MfE

Edition 12 2018 19

Link to Contents

Introduction

Formula Handbook

including
Mathematics,
Statistics,
Computer Algebra,
and
Engineering
Formulae

Download an electronic version of this handbook (with working hyperlinks) using the URL below.

http://is.gd/Formula_Handbook - pdf

Name_____

Course_____

Introduction

This handbook was designed to provide engineering students at Aberdeen College with the formulae required for their courses up to Higher National level (2nd year university equivalent).

The interactive graphs can be used online with the Geogebra app available in Google. Also, you can download Geogebra (see 3).

I have the handbook copied as an A5 booklet with a spiral binding. The covers are printed on thin card rather than paper. See Handbook Layout

It is typed in LibreOffice Writer using Liberation Sans font for text and Linux Libertine for formulae. Many of the illustrations are imported from Geogebra (see 3). Future developments will include more hyperlinks within the handbook and to other maths sites.

Any contributions will be gratefully accepted and acknowledged in the handbook.
If you prefer, you can make changes or add to the handbook within the terms of the Creative Commons licence. I will send you a copy the original LibreOffice file on request but please send me a copy of your work and be prepared to have it incorporated or adapted for inclusion in my version.
My overriding concern is for the handbook to live on and be continuously improved.
I hope that you find the handbook useful and that you will enjoy using it and that that you will feel inspired to contribute material and suggest hyperlinks that could be added.

Many thanks to my colleagues at North East Scotland College (formally Aberdeen College) and elsewhere for their contributions and help in editing the handbook. Special thanks are due to Mark Perkins at Bedford College who helped with the format and contributed to the contents.

If you find any errors or have suggestions for changes please contact the editor:
Peter K Nicol. (peterknicol@gmail.com)
or Michael Tamburrini (mick.tamburrini@gmail.com) **Contents**

Peter K Nicol
Aberdeenshire,
Scotland

12th Edition VI/MMXVIII

26/12/18
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A quote from Albert Einstein: **[I do not] carry such information in my mind since it is readily available in books. ..The value of a college education is not the learning of many facts but the training of the mind to think.**
In response to not knowing the speed of sound as included in the Edison Test: *New York Times* (18 May 1921); *Einstein: His Life and Times* (1947) Philipp Frank, p. 185; *Einstein, A Life* (1996) by Denis Brian, p. 129; "Einstein Due Today" (February 2005) edited by József Illy, Manuscript 25-32 of the Einstein Paper Project; all previous sources as per *Einstein His Life and Universe*(2007) by Walter Isaacson, p. 299. Wikiquotes 7-11-17.

The complete URL:

https://drive.google.com/file/d/0BxVFFxKqVrNjdzZ1ZDFLRnhNcVU/edit?usp=sharing

All my links are here: Handbook Links

Contents

1 Recommended Books

referred to by author name in this handbook

1.1 Maths

General pre-NC and NC : **Countdown to Mathematics**; Graham and Sargent
Vol. 1 ISBN 0-201-13730-5, Vol. 2 ISBN 0-201-13731-3

NC **Foundation Maths**, Croft and Davison
ISBN 0-131-97921-3

NC and HN and Degree**:**
K Singh **Engineering Mathematics through Applications**;
Kuldeep Singh, ISBN 0-333-92224-7. (1st Edition) **(1)**
978-0-230-27479-2 (2nd Edition) **(2)**
www.palgrave.com/science/engineering/singh

Engineering Mathematics, 6th Edition, J Bird
ISBN 1-8561-7767-X

HN and degree: **Higher Engineering Mathematics**, 4th Edition, J Bird,
J Bird ISBN 0-7506-6266-2

Degree **Engineering Mathematics** 6th Edition , K A Stroud
ISBN 978-1- 4039-4246-3

1.2 Mechanical and Electrical Engineering

NC **Advanced Physics for You**, K Johnson, S Hewett et al.
ISBN 0 7487 5296 X

Science for Engineering, 5th Edition, John Bird
ISBN 113882688X

Mechanical Engineering

NC and HN **Mechanical Engineering Principles**, C Ross, J Bird
ISBN 0750652284

Electrical Engineering

NC and HN **Basic Electrical Engineering Science**
Ian McKenzie Smith, ISBN 0-582-42429-1

HN **Power Electronics**
Cyril W Lander, ISBN 0 077 077 148

Note: Many of the books above have earlier editions which are just as good but are available at very reasonable prices online.

2 Useful Web Sites

If you use any of the sites below please read the instructions first. When entering mathematical expressions **the syntax MUST be correct**. See section 18 of this book. Most sites have examples as well as instructions. It is well worth trying the examples first. If you find anything really useful in the sites below or any other site please tell us so that we can pass on the information to other students.

Maths for Engineering	Michael Tamburrini's website contains notes practice material and links for many of the SQA Engineering maths units. **http://www.mathsforengineering.com/**
Notes from the Board	Everything we have written on the board for various units over the last year or so **https://is.gd/Petersnotes** **https://is.gd/Michaelsnotes**
Maths for Engineering NESCol	All the notes and practice material for the SQA Maths units taught as part of the level 4 to level 8 engineering courses. **http://bit.ly/mathsforengineering**
Mathcentre **MC**	Try the **Video Tutorials**. **http://www.mathcentre.ac.uk** The other stuff is excellent too. Also see **http://www.mathtutor.ac.uk**
Khan Academy **Khan**	The "free classroom of the World" Many video lectures using a blackboard **http://www.khanacademy.org**
WolframAlpha	Almost any maths problem solved! **http://www.wolframalpha.com/**
Derivative Calculator	Even shows the working (beta). Uses Maxima CAS **http://www.derivative-calculator.net** Recommended by Arthur Murphy
Integral Calculator	**www.integral-calculator.com**
BetterExplained **BE**	It is true – maths and some other topics explained better. **http://BetterExplained.com/** how to learn maths **how to learn maths**
Learnerstv.com	Draws together many Khan Academy, US videos, lecture notes and animations. Includes Maths and Statistics and Engineering sections. Also see Physical animations **http://www.learnerstv.com/index.php**
Technology Student	Recommended for all aspects of practical engineering **http://www.technologystudent.com/**
Efunda	A US service providing a wealth of engineering information on materials, processes, **Maths**, unit conversion and more. Excellent calculators (like quickmath). **http://www.efunda.com**
Wonderful Engineering	An engineering magazine that brings new, interesting, popular and innovative engineering news. Recommended by Paul McKenzie **http://www.*wonderfulengineering.com/*** **https://en-gb.facebook.com/WonderfulEngineering/**

QuickMath Links you to a computer running MATHEMATICA - the most powerful mathematical software.
http://www.quickmath.com

Math Insight The Math Insight web site is a collection of pages and applets designed to shed light on concepts underlying a few topics in mathematics. http://mathinsight.org

Mathway Try the problem solver for algebra, trig and calculus and it draws graphs too. See 18 for input syntax.
http://www.mathway.com

The Open University There are a lot of excellent courses to study and if you want to improve your maths I suggest that you start here
http://mathschoices.open.ac.uk/
Read the text very carefully on all the pages and then go to **http://mathschoices.open.ac.uk/routes/p6/index.html** and try the quizzes.

OU Learning Space Many courses for many levels - try them
http://openlearn.open.ac.uk/course/category.php?id=8&perpage=15&page=1
The one below is really useful

Using a Scientific Calculator Do you have a Casio fx-83 ES scientific calculator (or a compatible model) and want to learn how to use it? This unit will help you to understand how to use the different facilities and functions and discover what a powerful tool this calculator can be!
http://openlearn.open.ac.uk/course/view.php?name=MU123_1

Plus Magazine Plus magazine opens a door to the world of maths, with all its beauty and applications, by providing articles from the top mathematicians and science writers on topics as diverse as art, medicine, cosmology and sport. You can read the latest mathematical news on the site every week, browse our blog, listen to our podcasts and keep up-to-date by subscribing to Plus (on email, RSS, Facebook, iTunes or Twitter).
http://plus.maths.org/content/

Paul's Online Math Notes http://tutorial.math.lamar.edu/

Math10.com Worth exploring http://www.math10.com

The Narrow Road Maths Explained by Leland McInnes
http://zenandmath.wordpress.com/

Casio Calculator Manuals (in pdf format)
http://support.casio.com/manual.php?rgn=2&cid=004

If you come across any Engineering or Mathematics sites that might be useful to students on your course please tell me (Peter Nicol) - peterknicol@gmail.com

3 Recommended Computer Programs

wxMaxima  free (Open Source) MS Windows, Mac and Linux
http://andrejv.github.io/wxmaxima/index.html

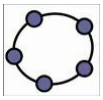
MS Windows installation: https://themaximalist.org/download-and-install-maxima/
MacOS installation: https://themaximalist.org/about/my-mac-os-installation/

See wxMaxima Introduction at Notes and exercises
See http://www.math.hawaii.edu/~aaronts/maximatutorial.pdf a simple introduction.
See http://www.neng.usu.edu/cee/faculty/gurro/Maxima.html but put in expression first!
and Maxima by Example http://www.csulb.edu/~woollett/
A open source free download computer algebra system. It is being constantly updated.
You are **not** allowed implicit multiplication.

$5\,e^{2\,t}+3\sin\left(\frac{\pi}{4}\right)$ typed as $5*\exp(2*t)+3*\sin(\%\,\mathrm{pi}/4)$

The % sign designates special functions. (numerical values of letters)

Maxima is a system for the manipulation of symbolic and numerical expressions, including differentiation, integration, Taylor series, Laplace transforms and ordinary differential equations. Also, Maxima can plot functions and data in two and three dimensions.

Geogebra  free (Open Source) MS Windows and Linux
http://www.geogebra.org

This program can be accessed over the web i.e. you do not need to download it although you need to be running Java Runtime Environment (free download). GeoGebra is a dynamic mathematics software that joins geometry, algebra and calculus. An expression in the algebra window corresponds to an object in the geometry window and vice versa.

Mathcad Commercial Software MS Windows
This is the tool of choice for most engineering mathematics. Notes available.
Mathcad Notes

Applied Maths

Definition of variables and functions
variable := number and units (:= use colon **:**)

Example: x:3kg will read as $x:=3\,\mathrm{kg}$ and a:5 m/s^2 as $a:=5\,\frac{m}{s^2}$

Function $f(x)$:= function in terms of x

Example: f(x): x*a will be interpreted as $f(x):=x\cdot a$
= gives numerical answer

Example $f(x)=$ will produce the answer $15\,N$

You can type a different unit in place of the box and the number will change to satisfy the units chosen.

- -

Symbolic Maths

$f(x)$ = use Boolean **(bold)** equals
→ symbolic units

Implicit multiplication: This is allowed but only with variables that cannot be confused with units.
For example, $3\,x$ is fine but $3\,s$ must be typed as $3*s$.
When editing expressions use the Ins key to change from editing to the left to editing to the right of cursor.

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4 Computer Input

wxMaxima and Geogebra are recommended .
Most of this also applies to spreadsheets and online maths sites.
Spreadsheet programs are not recommended (except for statistical calculations).

Calculator key	Computer (Keyboard) entry			
	Geogebra (3)	Mathcad (2)	Spreadsheet (6)	wxMaxima (5)
$\times$	*	* (Shift 8)	*	*
$\div$	/	/	/	/
x^2	^2	^2 (Shift 6 then 2)	#^2	^2 (Shift 6 then 2)
$x^{\blacksquare}$ or ^ or x^y or y^x	^	^	^(1/#)	^
$\sqrt{\ }$	**sqrt**() (also on drop down list)	\	**sqrt(#)**	**sqrt( )**
$\sqrt[x]{\ }$	^ $\frac{1}{x}$	$\sqrt[n]{\ }$ Calculator toolbar	^ $\frac{1}{\#}$	^ $\frac{1}{x}$
$5\sin(x^o+30^o)$ (1)	$5\sin(x^o+30^o)$ o symbol from drop down list	$5\sin(x\ deg+30\ deg)$	5*sin(x/180*pi()+ 30/180*pi())	5*sin(x/180* %pi+ 30/180* %pi)
e^x	e from drop down list then ^ or exp( )	e^x	exp(#)	exp( ) or %e^( )
$\ln$	$\ln$	$\ln$	LN(#)	log( )
π	pi	p CTRL g	pi()	%pi
$10\times\pi\times0.7$	10 pi *0.7	10 CTRL g*0.7	10*pi()*0.7	10*%pi*0.7
$\sin^{-1}(0.5)$ means arcsin(0.5)	asin(0.5)	asin(0.5)	asin(#)	asin(0.5)

(1) As all programs work in radians by default you must change every input into degrees (if you have to work in degrees).
(2) Also available on toolbars.
(3) Only x allowed as variable
(4) See also 14.5
(5) In wxMaxima typing pi will produce π as a variable NOT 3.1415...
The same is true for e .
(6) # indicates cell address

Back to **2 Web Sites**

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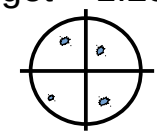
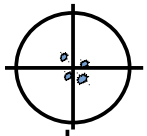
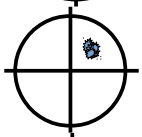
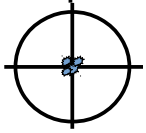
5 Evaluation

5.1.1 Accuracy and Precision

see 25

Example: Target = 1.234 - 4 possible answers

Not Accurate, not Precise		1.270, 2.130, 0.835, 1.425
Accurate but not Precise		1.231, 1.235, 1.232, 1.236
Precise but not Accurate		1.276, 1.276, 1.276. 1,276
Precise and Accurate		1.234, 1.234, 1.234, 1.234

5.1.2 Units

MC

Treat units as algebra - see Metre Cube

for example $KE=\frac{1}{2}m\,v^2$ where $m=5\,\text{kg}$ and $v=12\frac{\text{m}}{\text{s}}$.

$$KE=\frac{1}{2}\times 5\times\text{kg}\times 12\times\left(\frac{\text{m}}{\text{s}}\right)^2$$

$$KE=\frac{1}{2}\times 5\times\text{kg}\times 12^2\times\frac{\text{m}^2}{\text{s}^2}$$

$$KE=\frac{1}{2}\times 5\times 12^2\times\frac{\text{kg}\times\text{m}^2}{\text{s}^2}$$

$$KE=360\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

$KE=360\,\text{J}$

Standard workshop tolerance ± 0.2 mm

5.1.3 Rounding

Do not round calculations until the last line.
Round to significant figures preferably in engineering form

Example: $A=\frac{\pi d^2}{4}$ where $d=40$

$A=1256.637061$ $A=1.256637061\times 10^3$

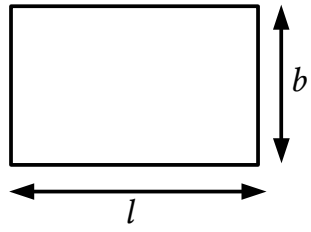
$A=1.257\times 10^3$ rounded to 4 sig fig ($A=1257$)

There should be **at least** 2 more significant figures in the calculation than in the answer.

5.1.4 Square Root Algorithm

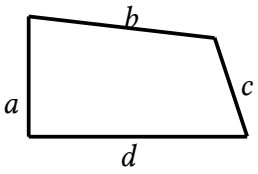
Guess root of N is $x_1=Ans$ then $\frac{\left(\frac{N}{Ans}+Ans\right)}{2}=x_2=x_3=\ =$ until $x_{n-1}=x_n$

6 Areas and Volumes

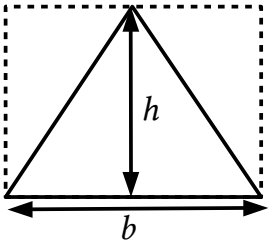


Rectangle $A = l\,b$

$P = 2(l + b)$ (perimeter)

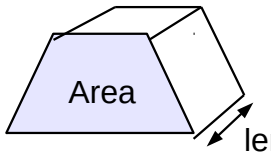


$P = a + b + c + d$



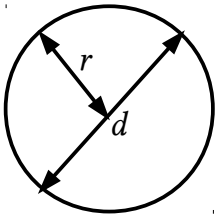
Triangle $A = \frac{1}{2}\,b\,h$

Volume of a Prism



Volume = Area x length

(Uniform cross sectional area)



Circle $A = \frac{\pi d^2}{4}$ $A = \pi r^2$

$C = \pi d$ $C = 2\pi r$

π – use π button

$\pi \approx \frac{355}{113}$ $\pi \approx 3.14159$

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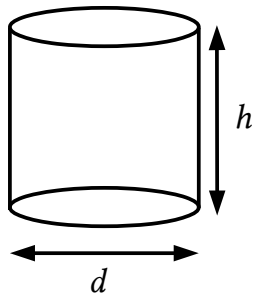
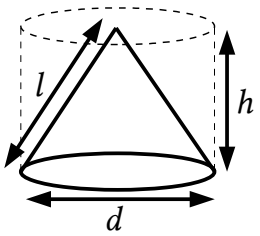


Cylinder V = Area of circular base times height

$V_{cyl} = \frac{\pi d^2}{4} \times h$ $V_{cyl} = \pi r^2 \times h$

Total **surface** area = $A = \pi d\,h + 2\left(\frac{\pi d^2}{4}\right)$ $A = 2\pi r\,h + 2\pi r^2$

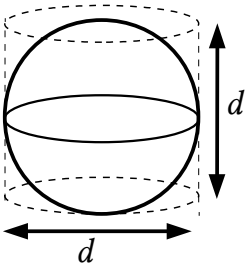
(side + 2 ends)



Cone $V_{cone} = \frac{V_{cyl}}{3}$ or $V = \frac{\pi d^2 h}{12}$ $V = \frac{\pi r^2 h}{3}$

Curved surface area $A = \frac{\pi d\,l}{2}$ $A = \pi r\,l$

Total surface area $A = \pi r\,l + \pi r^2$



Sphere $V_{sphere} = \frac{2\,V_{cyl}}{3}$ $V = \frac{\pi d^3}{6}$ $V = \frac{4\pi r^3}{3}$

Total surface area $A = \pi d^2$ $A = 4\pi r^2$

7 Mathematical Notation – what the symbols mean

see 23.1.1 MC

$\in$	is a member of. ($x \in \mathbb{R}$ means x is a member of $\mathbb{R}$)
$\subset$	is a subset of. ($B \subset A$ means set B is a subset of set A)
$\mathbb{N}$	the set of natural numbers 1, 2, 3, ...
$\mathbb{Z}$	the set of all integers ....., -2, -1, 0, 1, 2, 3, ...
$\mathbb{Q}$	the set of rational numbers including $\mathbb{Z}$ and fractions $\frac{p}{q}; p, q \in \mathbb{Z}$
$\mathbb{R}$	the set of all real numbers. Numbers represented by drawing a continuous number line.
$\mathbb{C}$	the set of complex numbers. Numbers represented by drawing vectors.
	$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$
$\therefore$	therefore
w.r.t.	with respect to
*	used as a multiplication sign ($\times$) (in computer algebra)
^	used as "power of" (x^y) in computer algebra
$\neq$	not equal to
$\approx$	approximately equal to
>	greater than. $x > 2$ means x is greater than and not equal to 2
$\geq$	greater than or equal to.
<	less than. $a < 2$ means a is less than and not equal to 2.
$\leq$	less than or equal to.
$a \leq x \leq b$	x is greater than or equal to a and less than or equal to b
ab	abbreviation for $a \times b$ or $a * b$ or $a \cdot b$
$a \times 10^n$	a number in scientific (or standard) form. ($3 \times 10^3 = 3000$)
	use EXP or $\times 10^x$ key on a calculator
$n!$	" n factorial" $n \times (n-1) \times (n-2) \times (n-3) \times ... \times 1$
$A \propto B$	implies $A = k B$ where k is a constant (direct variation)
$\lvert x \rvert$	the modulus or absolute value of x . Abs on a calculator. The magnitude of the number x irrespective of the sign. $\lvert -3 \rvert = 3 = \lvert 3 \rvert$
∞	infinity
$\Rightarrow$	implies

Repeated or recurring decimal $\frac{1}{3} = 0.33333.... = 0.\dot{3} = 0.(3) = 0.\overline{3}$,

$\frac{2}{7} = 0.285714.... = 0.\dot{2}8571\dot{4} = 0.(285714) = 0.\overline{285714}$

7.1.1 Notation for Indices and Logarithms

MC

a^n — abbreviation for $a \times a \times a \times a ... \times a$ (n terms). see 17.1
$x^{\blacksquare}$ or ^ or x^y or y^x or a^b on a calculator.

$\sqrt{a}$ — the positive square root of the number a. $\sqrt{x} = x^{\frac{1}{2}} = x^{0.5}$

$\sqrt[k]{a}$ — k th root of a number a. $\sqrt[3]{8} = 2$ $\sqrt[k]{a} = a^{\frac{1}{k}}$.

e^x — $\exp(x)$ (2.71828.... to the power of x). See 17.1.

$\log_e(x)$ — $\ln(x)$ on a calculator. The logarithm of x to the base e

$\log_{10}(x)$ — $\log(x)$ on a calculator. The logarithm of x to the base 10

- -

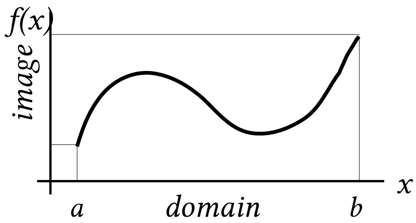
7.1.2 Notation for Functions

$f(x)$ — a function of x. Also seen as $g(x)$, $h(x)$, $y(x)$

$f^{-1}(x)$ — the inverse of the function labelled $f(x)$ MC

$g \circ f$ — the composite function - first f then g or $g(f(x))$. MC

Domain and Image



***Note**: Range can be used instead of image but it can be confused with the statistical meaning. see 19.1.1*

$\{x : x \in \mathbb{R}, a \leq x \leq b\}$ Set with members x such that (:) x is in set $\mathbb{R}$ and x is greater than or equal to a and less than or equal to b.

$\{x : x \in \mathbb{R}, a < x < b\}$ Set with members x such that (:) x is in set $\mathbb{R}$ and x is greater than a and less than b.

Asymptote — A horizontal asymptote is an output value y that a function never quite reaches. $f(x) < y$
A vertical asymptote is an input value a which is never quite reached $x < a$ or results in an undefined value for the function (dividing by 0).

See 17.3 and 17.4

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8 Laws of Mathematics

Associative laws - for addition and multiplication

$a+(b+c)=(a+b)+c$ $\qquad$ $a(bc)=(ab)c$

Commutative laws - for addition and multiplication

$a+b=b+a$ but $a-b\neq b-a$

$ab=ba$ but $\frac{a}{b}\neq\frac{b}{a}$

Distributive laws - for multiplication and division

$a(b+c)=ab+ac$ $\qquad$ $\frac{b+c}{a}=\frac{b}{a}+\frac{c}{a}$

Arithmetical Identities

$x+0=x$ $\qquad$ $x\times 1=x$ $\qquad$ $(x\times 0=0)$

Algebraic Identities K Singh pp 73 – 75

$(a+b)^2=(a+b)(a+b)=a^2+2ab+b^2$ $\qquad$ $a^2-b^2=(a+b)(a-b)$

$(a+b)^3=(a+b)(a^2+2ab+b^2)=a^3+3a^2b+3ab^2+b^3$ see 17.4

Other useful facts

$a-b=a+(-b)$ $\qquad$ $\frac{a}{b}=a\div b=\frac{a}{1}\times\frac{1}{b}$

$a-(-b)=a--b=a+b$

$\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$ $\qquad$ $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$ see 17.3.9, 24

$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$ **MC**

$(a+b)(c+d)=ac+ad+bc+bd$ FOIL **MC**

8.1.1.1 Proportionality

A is directly proportional to B $\qquad$ $A\propto B$ $\qquad$ $A=kB$

A is inversely proportional to B $\qquad$ $A\propto\frac{1}{B}$ $\qquad$ $A=\frac{k}{B}$

Procedure: Given values for A and B find the value of k.
Write formula replacing k with its value

8.2 Algebra

8.2.1 Sequence of operations

[K Singh (1) 40-43 (2) 40-43]

Sequence of operations - the same sequence as used by scientific calculators.

Brackets	$(\quad)$	*come before*
Of	x^2, $\sqrt{x}$, $\sin x$, e^x, *"square **of** x , sine **of** x*	*comes before*
Division	$\div$	*comes before*
Multiplication	$\times$	*comes before*
Addition	+	*comes before*
Subtraction	$-$	

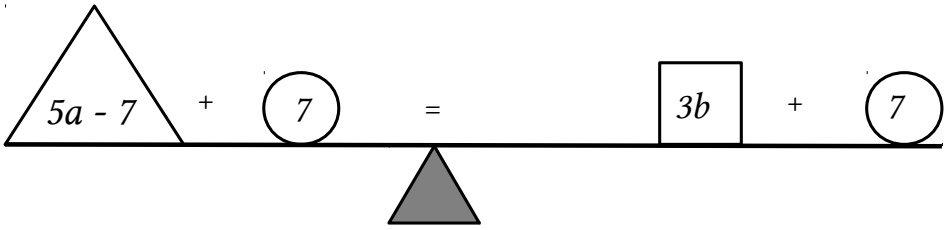
$y = 3\sin(ax^2 + b) - 5$ would be read in this order

	"Do"	"Undo"
left bracket		
x	x	x
x squared	x^2	$\sqrt{(\ )}$
times a	$\times a$	$\div a$
plus b	$+ b$	$- b$
right bracket		
sine **of** the result ($\sin(ax^2 + b)$)	$\sin(\ )$	$\sin^{-1}(\ )$
times 3	$\times 3$	$\div 3$
minus 5	$- 5$	$+ 5$
equals y	y	y

8.2.2 Changing the subject of a Formula

[K Singh (1) 53-66 (2) 53-56]

An equation or formula must always be **BALANCED** - **whatever** mathematical operation you do to one side of an equals sign must be done to other side as well. (to **all** the terms)



You can't **move** a term (or number) from one side of the equals sign to the other, you must **UNDO** it by using the correct **MATHEMATICAL** operation.

UNDO	$+$	with	$-$	**and**	$-$	with	$+$	
UNDO	$\times$	with	$\div$	**and**	$\div$	with	$\times$	
UNDO	$\sqrt{\ }$	with	x^2	**and**	x^2	with	$\sqrt{\ }$	
UNDO	x^n	with	$\sqrt[n]{\ }$	**and**	$\sqrt[n]{\ }$	with	x^n	
UNDO	$\sin x$	with	$\sin^{-1} x$	**and**	$\sin^{-1} x$	with	$\sin x$	
UNDO	e^x	with	$\ln x$	**and**	$\ln x$	with	e^x	(ln button)
UNDO	10^x	with	$\log_{10} x$	**and**	$\log_{10} x$	with	10^x	(log button)
UNDO	$\frac{dy}{dx}$	with	$\int dx$	**and**	$\int dx$	with	$\frac{dy}{dx}$	
UNDO	$\mathscr{L}[y]$	with	$\mathscr{L}^{-1}[y]$	**and**	$\mathscr{L}^{-1}[y]$	with	$\mathscr{L}[y]$	

etc

A term is joined to another term by a + or - sign

Generally (but not always) start with the terms **FURTHEST AWAY** from the new subject **FIRST.**

Think of the terms in the formula as layers of an onion - take the layers off one by one.

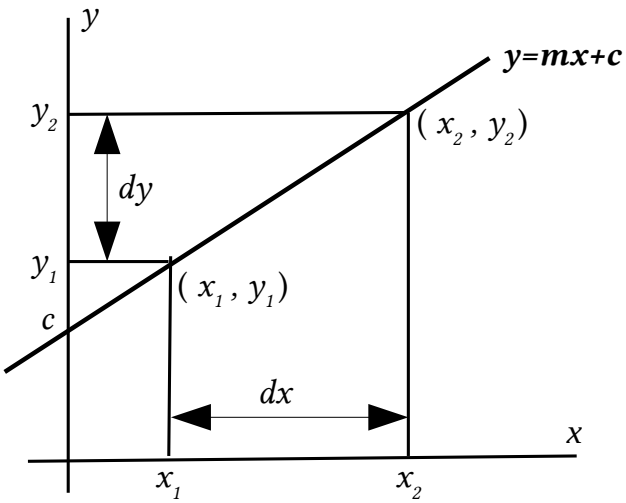
$$((a((x^2)))+b)$$

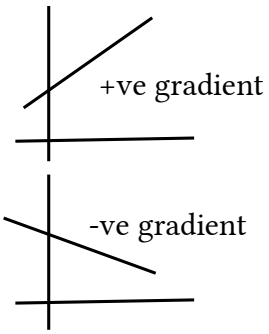
Try this first http://www.waldomaths.com/Equation3NLW.jsp
There is a link to Equations 1 if you need a bit more help.

MC

9 The Straight Line

[K Singh (1) 100–108 (2) 101-110]





The general equation of a straight line is $y=mx+c$ 18.1.1, 18..2 ,14.3

It has gradient of $m=\frac{(y_2-y_1)}{(x_2-x_1)}$ or $\frac{dy}{dx}=\frac{(y_2-y_1)}{(x_2-x_1)}$ and it cuts the y axis at $(0,c)$

or $y_1=mx_1+c$ (1)

$y_2=mx_2+c$ (2) then (1) – (2) and solve for m (then c)

Also: A straight line, gradient m passing through (a,b) has the equation:

$(y-b)=m(x-a)$ Also see 18.2.3, 18.3.10, 19.2.1, 18.4.5, 11, **MC**

9.1.1 Find the 'best fit' formula for a set of data

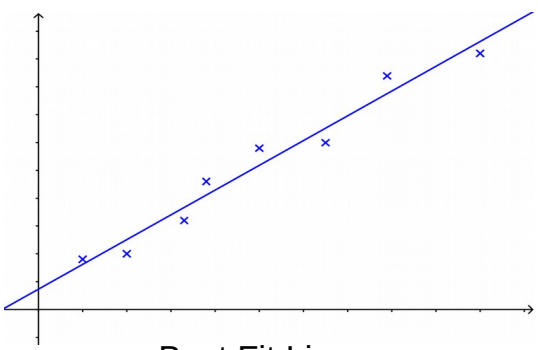
see 4 and 19.2.1

Data presented as

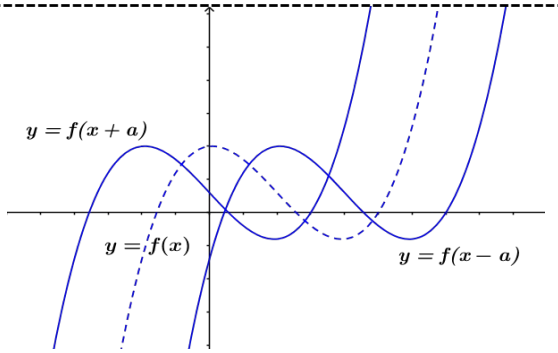
x	x_1	x_2	x_3	x_4	etc
y	y_1	y_2	y_3	y_4	etc

Basic Procedure:-

	Put **x** data in column A and **y** data in column B
Highlight	All data
Select	Insert Chart (or chart symbol)
Chart Type	XY (Scatter)
Titles	Give graph and axes titles (remove Legend)
Right click on data point	Add Trendline
Type	Choose most appropriate (best fit to data)
Options	Display equation on chart (best fit formula)
Options	Display R^2 value on chart ($0.95 \le R^2 \le 1$)
Rt click on trendline	Change type to get R^2 nearer 1 (perfect fit)



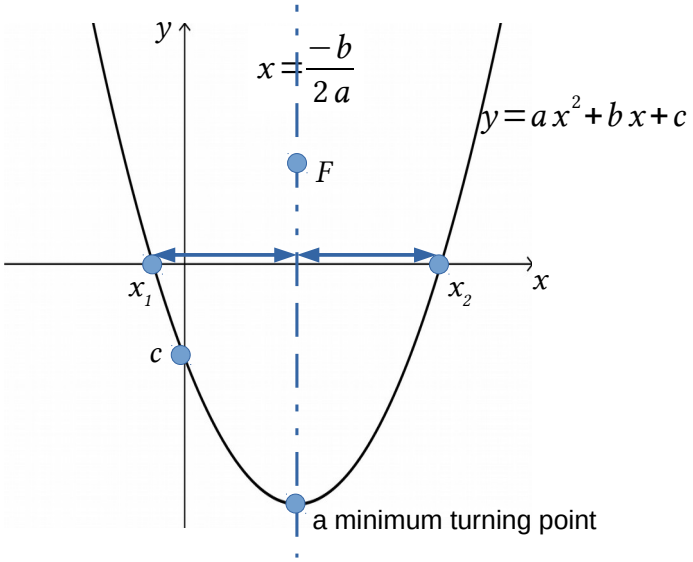
Best Fit Line

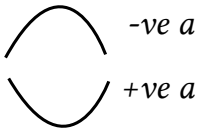


Graph Translation

10 Quadratic Equations

[K Singh (1,2) 86 - 90 & (1) 109 - 113 (2) 110-113]





Focus $F=\left(\frac{-b}{2a},\frac{-(b^2-4ac-1)}{4a}\right)$

Geogebra quadratic MC

The real solutions (roots) x_1 and x_2 of the equation $ax^2+bx+c=0$ are the value(s) of x where $y=ax^2+bx+c$ crosses the x axis.

The solutions (roots) x_1 and x_2 of $ax^2+bx+c=0$ are given by the Quadratic Formula.

$$x=\frac{-b}{(2a)}\pm\frac{\sqrt{(b^2-4ac)}}{(2a)} \quad \text{or} \quad x=\frac{\left(-b\pm\sqrt{(b^2-4ac)}\right)}{(2a)}$$

Definition of a root: The value(s) of x which make y equal to zero.

.....

Also:

$$ax^2+bx+c=0$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$\left(x+\frac{\left(\frac{b}{a}\right)}{2}\right)^2+d^2=0$$

where $d^2=\frac{c}{a}-\left(\frac{\left(\frac{b}{a}\right)}{2}\right)^2$ see 18.4.3

If $y=k(x+A)^2+B$ the turning point is $(-A,B)$ Geogebra

---

11 Simultaneous Equations with 2 variables

[K Singh (1) pp 90-98 (2) 90-99]

General method:

Write down both equations and label (1) and (2).

$ax+by=e \quad (1)$

$cx+dy=f \quad (2)$

Multiply every term on both sides of (1) by c and every term on both sides of (2) by a and re-label as (3) and (4).

$cax+cby=ce \quad (3)$

$acx+ady=af \quad (4)$

Multiply every term on both sides of (4) by -1 and re-label.

$cax+cby=ce \quad (3)$

$-acx-ady=-af \quad (5)$

Add (3) to (5) to eliminate x

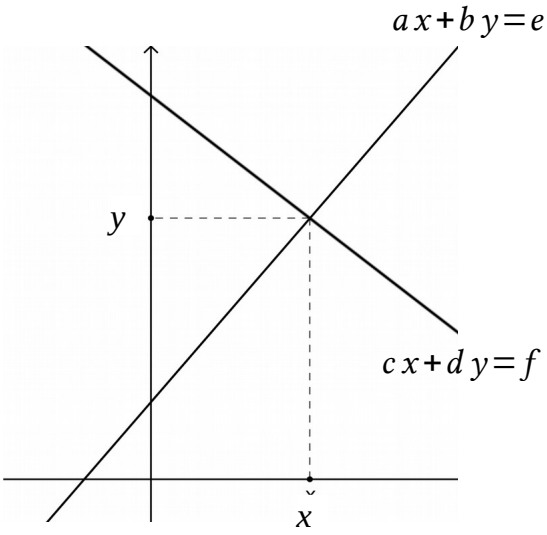
Calculate the value of y

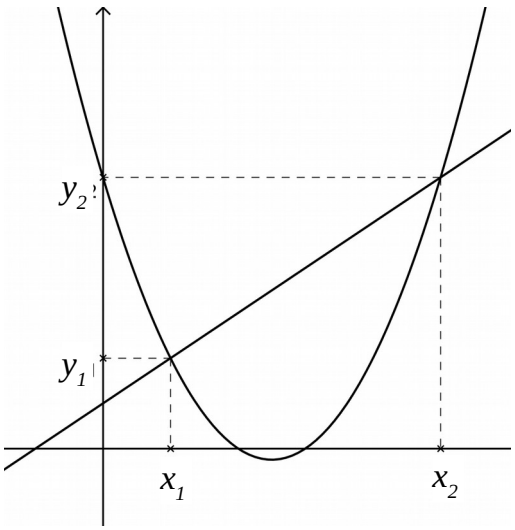
Substitute the value of y into equation (1)

Calculate the value of x **MC**

Check by substituting the values of x and y into (2)

Graphical Solution





If $f(x)=g(x)$ then $f(x)-g(x)=0$ - also see 9 and 10

---

12 Matrices

[K Singh (1) pp 507 – 566 (2) 560-635]

Notation:

Identity $I=\begin{pmatrix} 1 & 0 & 0 & .. \\ 0 & 1 & 0 & .. \\ 0 & 0 & 1 & .. \\ . & . & . & .. \end{pmatrix}$

A $m\times n$ matrix has m rows and n columns.

a_{ij} an element in the i th row and j th column.

If $A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and $B=\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

then $A+B=\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}$

Wiki

and $A\times B=\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \end{pmatrix}$ $\text{Columns}_A=\text{Rows}_B$

12.1.1.1 Solution of Equations 2 x 2

If $AX=B$ then $X=A^{-1}B$ $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} e \\ f \end{pmatrix}$

If $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

then the **inverse matrix**,

$$A^{-1}=\frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix},\ (ad-bc\neq 0)$$

MC

where the **Determinant** $\det A=\begin{vmatrix} a & b \\ c & d \end{vmatrix}=ad-bc$

- -

12.1.1.2 wxMaxima Matrix multiplication

$\begin{pmatrix} \ \end{pmatrix}\cdot\begin{pmatrix} \ \end{pmatrix}$ Use . Do not use $*$

- -

12.1.1.3 Eigenvalues and Eigenvectors

The eigenvalues, λ are found by solving $det(A-\lambda I)=0$ 18.4.4
and the eigenvectors, $\boldsymbol{u}$, from $(A-\lambda I)\boldsymbol{u}=0$ MC

12.1.1.4 *Inverse Matrix, 3 x 3 (and larger square matrices)*

Start with $\left(\begin{array}{ccc|ccc} a_{11} & a_{12} & a_{13} & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 1 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 1 \end{array}\right)$ carry out row operations to:

$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & b_{11} & b_{12} & b_{13} \\ 0 & 1 & 0 & b_{21} & b_{22} & b_{23} \\ 0 & 0 & 1 & b_{31} & b_{32} & b_{33} \end{array}\right)$ where $\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = A^{-1}$

12.1.1.5 *Determinant of a 3 x 3 matrix*

$$det\ A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

or use Sarrus' Rule as below

$$det\ A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{pmatrix}$$

$$detA = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{33}a_{21}a_{12}$$

12.1.1.6 *Inverse of a 3 by 3 matrices by using co-factors*

$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ $A^{-1} = \frac{1}{detA}(adjA)$ where $adjA$ is the **adjoint** (**adjunct)** matrix of A

$adjA = C^T$ where C^T = the transpose of the co-factors of A

Co-factors

$cf(a) = det\begin{pmatrix} e & f \\ h & i \end{pmatrix}$ $cf(b) = -det\begin{pmatrix} d & f \\ g & i \end{pmatrix}$ $cf(c) = det\begin{pmatrix} d & e \\ g & h \end{pmatrix}$ Be careful of place signs!

$cf(d) = -det\begin{pmatrix} b & c \\ h & i \end{pmatrix}$ $cf(e) = det\begin{pmatrix} a & c \\ g & i \end{pmatrix}$ $cf(f) = -det\begin{pmatrix} a & b \\ g & h \end{pmatrix}$ $\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$

$cf(g) = det\begin{pmatrix} b & c \\ e & f \end{pmatrix}$ $cf(h) = -det\begin{pmatrix} a & c \\ d & f \end{pmatrix}$ $cf(i) = det\begin{pmatrix} a & b \\ d & e \end{pmatrix}$

Co-factor Matrix $C = \begin{pmatrix} cf(a) & cf(b) & cf(c) \\ cf(d) & cf(e) & cf(f) \\ cf(g) & cf(h) & cf(i) \end{pmatrix}$

Then, transpose the Co-factor Matrix (rows to columns)

Adjoint (Adjunct) Matrix $C^T = \begin{pmatrix} cf(a) & cf(d) & cf(g) \\ cf(b) & cf(e) & cf(h) \\ cf(c) & cf(f) & cf(i) \end{pmatrix} = adjA$

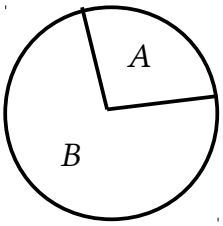
Thanks to Richard Kaczkowski, Calgary, Canada.

13 The Circle

13.1.1 Sectors and Segments

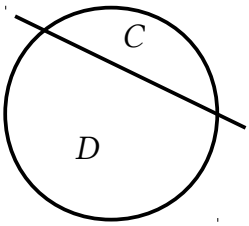
6

A Minor Sector



B Major Sector

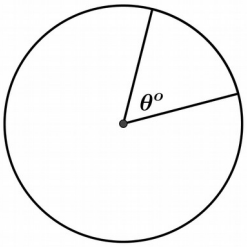
C Minor Segment



D Major Segment

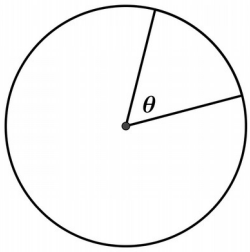
$$\text{Arc Length} = \frac{\theta^o}{360^o} 2\pi r$$

$$\text{Sector Area} = \frac{\theta^o}{360^o} \pi r^2$$



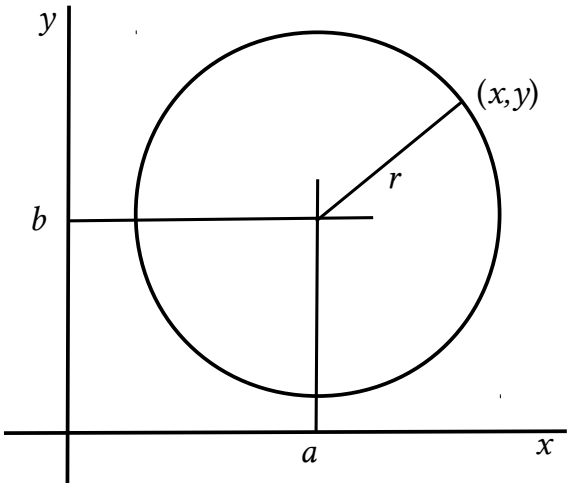
$$\text{Arc Length} = \theta r$$

$$\text{Sector Area} = \frac{\theta r^2}{2}$$



13.1.2 Circle Equations

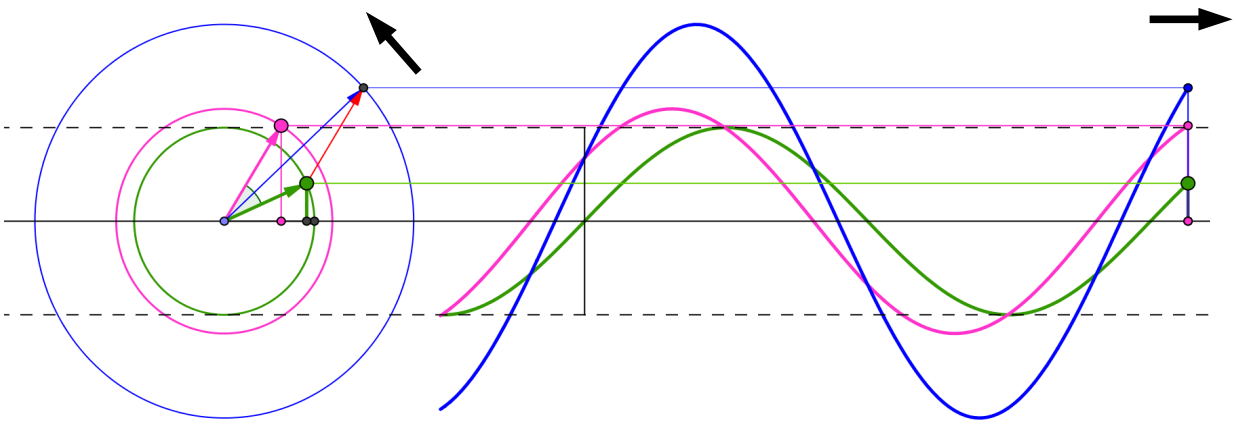
The equation $(x-a)^2+(y-b)^2=r^2$ represents a circle centre (a,b) and radius r.



Parametric $x=a+r\cos t, \quad y=b+r\sin t$

Generator and wave

Geogebra link

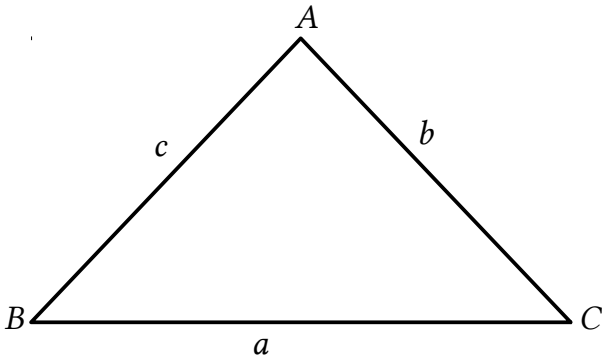


14 Trigonometry

[K Singh (1) 167-176 (2) 171-234]

14.1.1 Notation for Trigonometry

Labelling of a triangle



$\sin\theta$ the value of the sine function of the angle θ

$\cos\theta$ the value of the cosine function of the angle θ

$\tan\theta$ the value of the tangent function of the angle θ

$\theta=\sin^{-1}b$ $\arcsin b$ the value of the basic angle θ whose sine function value is b. $\left(-90^o\le\theta^o\le 90^o\right)$ or $\left(\frac{-\pi}{2}\le\theta\le\frac{\pi}{2}\right)$

$\theta=\cos^{-1}b$ $\arccos b$ the value of the basic angle θ whose cosine function value is b. $\left(0^o\le\theta^o\le 180^o\right)$ or $\left(0\le\theta\le\pi\right)$

$\theta=\tan^{-1}b$ $\arctan b$ the value of the basic angle θ whose tangent function value is b. $\left(-90^o\le\theta^o\le 90^o\right)$ or $\left(\frac{-\pi}{2}\le\theta\le\frac{\pi}{2}\right)$

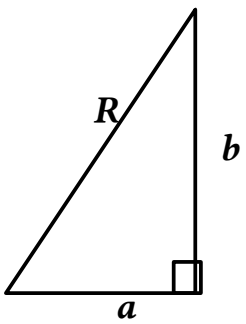
14.2 Pythagoras' Theorem

In a **right angled** triangle, with hypotenuse, length R, and the other two sides of lengths a and b, then

$$R^2=a^2+b^2$$

or

$$R=\sqrt{a^2+b^2}$$



use of Pythagoras' Theorem BE surprising uses
Pythagorean distance BE pythagorean distance
Interactive proof http://www.sunite.ubc.ca/LivingMathematics/V001N01/UBCExamples/Pythagoras/pythagoras.html

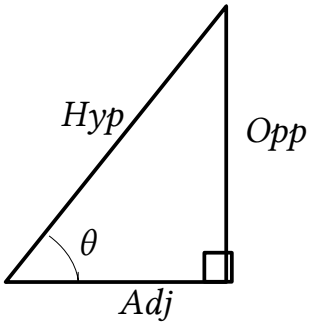
---

14.3 The Triangle

14.3.1 Trig Ratios

In a **right angled** triangle, with hypotenuse, (which is the longest side), of length Hyp ,

SOHCAHTOA



The other two sides have lengths Adj (adjacent, or next to angle θ) and Opp (opposite to angle θ) then

MC

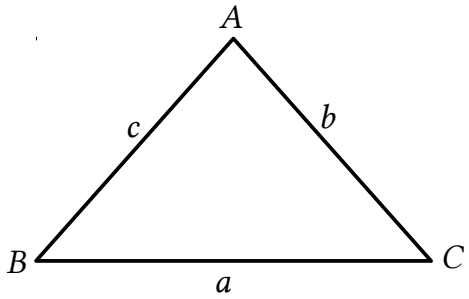
$$\sin(\theta)=\frac{Opp}{Hyp} \qquad \cos(\theta)=\frac{Adj}{Hyp} \qquad \tan(\theta)=\frac{Opp}{Adj}$$

see also 16.2 and 9

14.3.2 Sine and Cosine Rules and Area Formula

[K Singh (1) 187-192 (2) 195-191]

In **any** triangle ABC, where A is the angle at A, B is the angle at B and C is the angle at C the following hold:



Sine Rule

$$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$$

or $$\frac{\sin(A)}{a}=\frac{\sin(B)}{b}=\frac{\sin(C)}{c}$$ Khan

Cosine Rule

$$\cos(A)=\frac{(b^2+c^2-a^2)}{(2\,b\,c)}$$

or $$a^2=b^2+c^2-2\,b\,c\cos(A)$$ Khan

Area Formula

$$\text{Area} = \frac{b\,c\sin(A)}{2}$$

14.4 Trigonometric Graphs and Equations

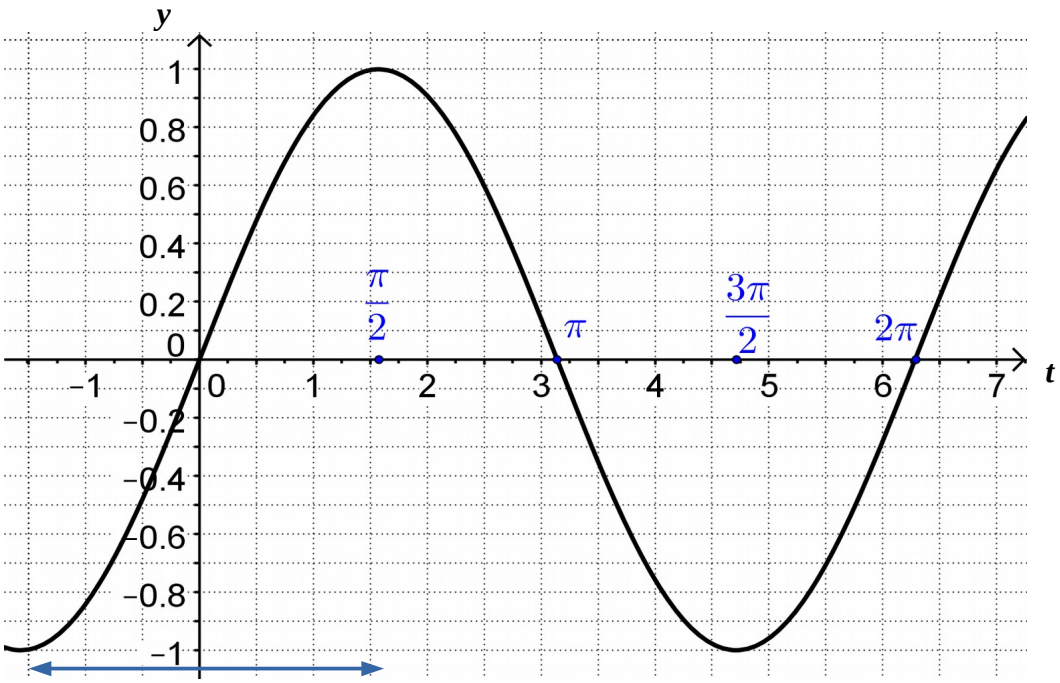
[K Singh (1) 177- 202 (2) 181- 210]

MC

14.4.1 Trigonometric Graphs – Radians

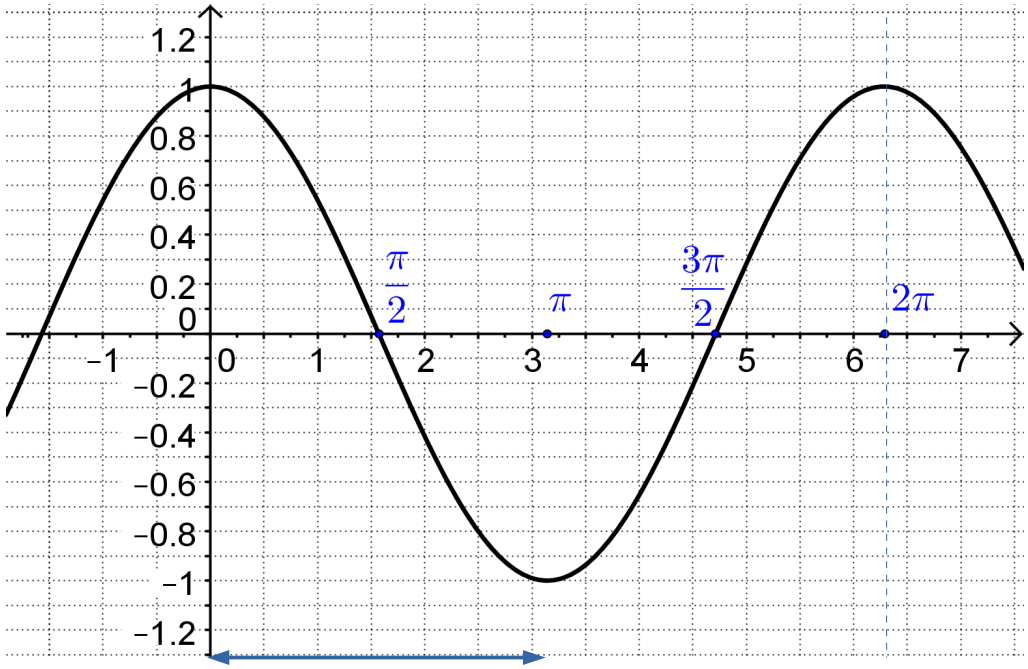
Radians i.e. no units - horizontal axis is usually time.

$y = \sin(t)$



Calculator answer

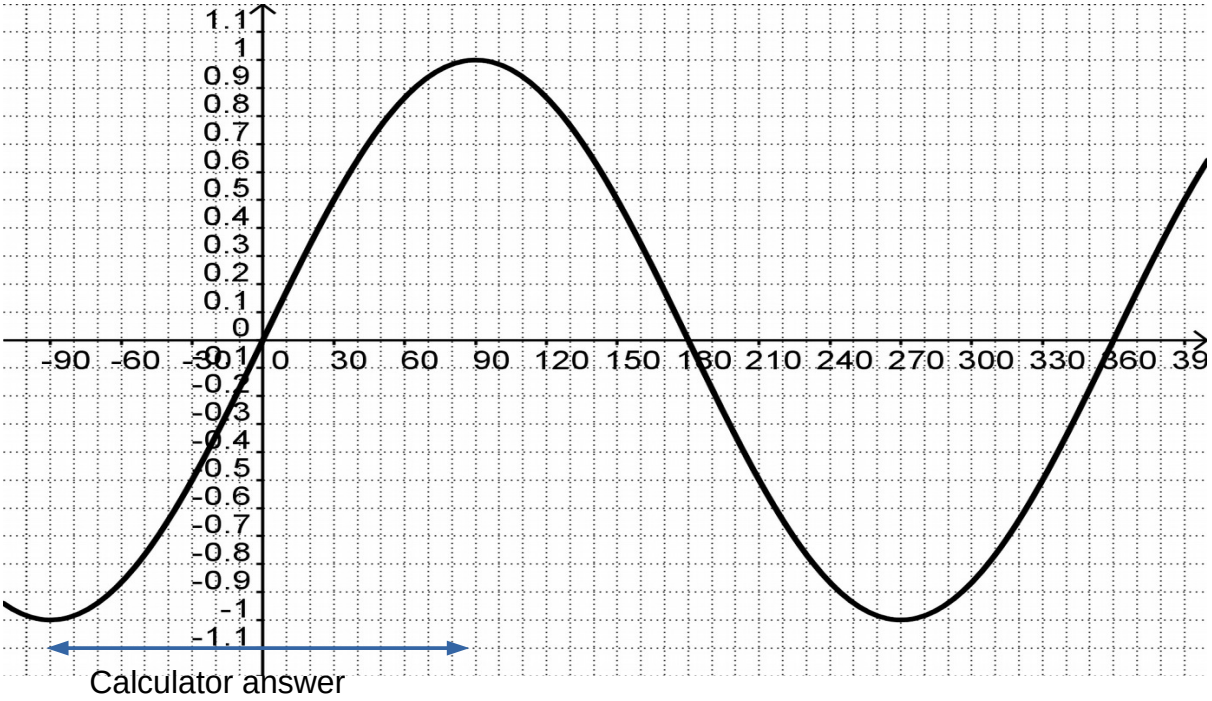
$y = \cos(t)$



Calculator answer

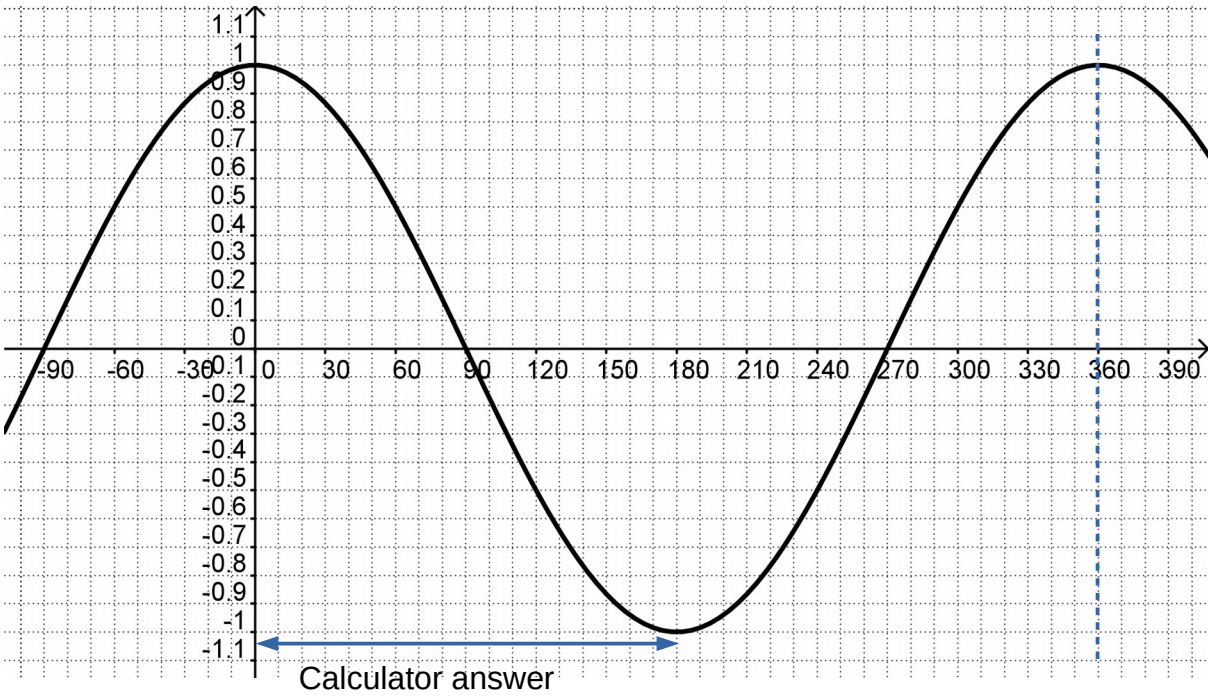
14.4.2 Trigonometric Graphs - Degrees

$$y = \sin(x^o)$$

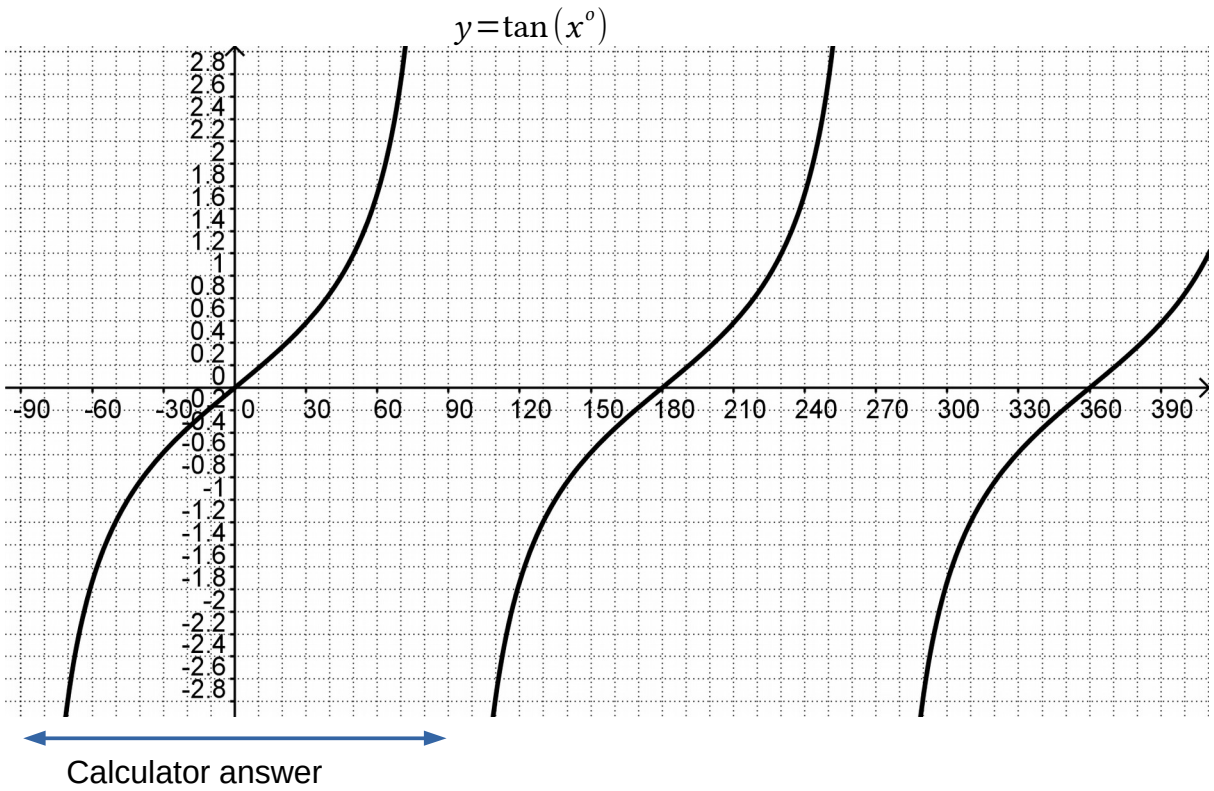


[Geogebra Sine wave slider]

$$y = \cos(x^o)$$



Geogebra Cosine wave slider



Calculator answer

14.4.3 Degrees - Radians Conversion

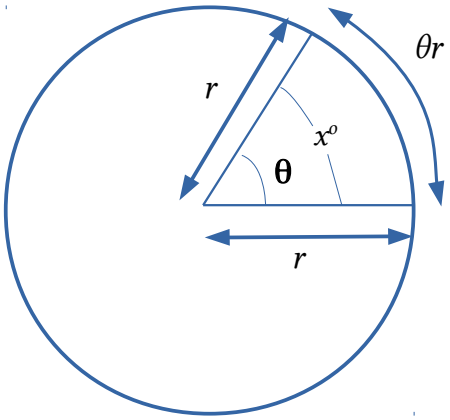
[K Singh (1) 192-195 (2) 201-204]

0, 30, 45, 60, 90, 120, 135, 150, 180, 210, 225, 240, 270, 300 315, 330, 360

$0 \quad \frac{\pi}{6} \quad \frac{\pi}{4} \quad \frac{\pi}{3} \quad \frac{\pi}{2} \quad \frac{2\pi}{3} \quad \frac{3\pi}{4} \quad \frac{5\pi}{6} \quad \pi \quad \frac{7\pi}{6} \quad \frac{5\pi}{4} \quad \frac{4\pi}{3} \quad \frac{3\pi}{2} \quad \frac{5\pi}{3} \quad \frac{7\pi}{4} \quad \frac{11\pi}{6} \quad 2\pi$

Degrees to radians $x^o \div 180 \times \pi = \theta \text{ rad}$

Radians to degrees $\theta \, rad \div \pi \times 180 = x^o$



Geogebra Radians

BE degrees and radians

see 22.1.2

A radian: A point travelling one radius length around the (outside) circumference of a circle has moved by the angle of one radian.
The angle θ subtended (or made by) an arc the same length as the radius of a circle. Notice that an arc is curved.

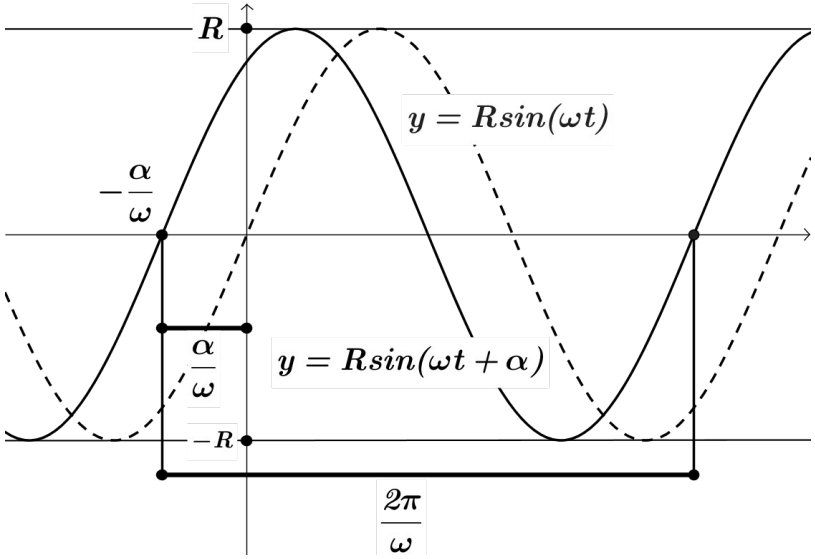
14.4.4 Solution of Trigonometric Equations

[K Singh (1) 177- 202 (2) 181- 210]

MC

14.4.5 Sinusoidal Wave

[K Singh (1) 195-202 (2) 204-212]



see 18.5, 24.4

Period $T = \frac{2\pi}{\omega}$

Phase angle $= \alpha$

Phase shift $= \frac{\alpha}{\omega}$

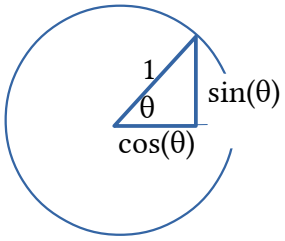
$\omega = 2\pi f$

$f =$ frequency

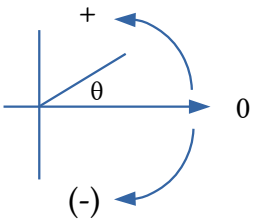
ω is the angular velocity in radii per second.

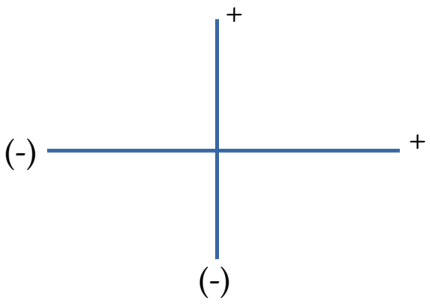
Notes and Exercises

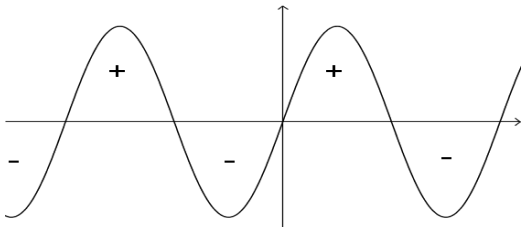
Thanks to Mark Perkins



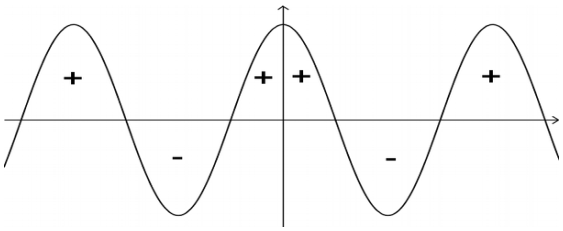
Unit Circle



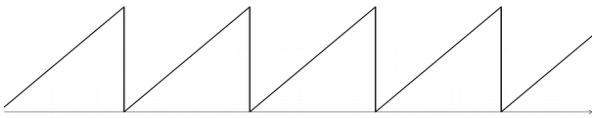




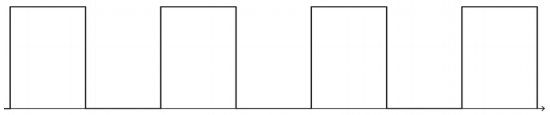
Odd Function



Even Function

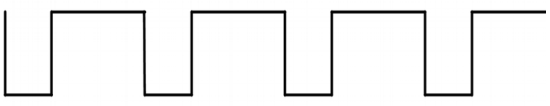


Saw Tooth



Square Wave

Triangular Wave



Pulse Wave

All periodic functions can be formed from a series of sine and cosine waves - see Fourier Transforms 18.5

14.5 Trigonometric Identities

[K Singh (1) 203-213 (2) 212-223]

$\tan(A)=\frac{\sin(A)}{\cos(A)}$ $\cot(A)=\frac{1}{\tan(A)}=\frac{\cos(A)}{\sin(A)}$, (the cotangent of A)

$\sec(A)=\frac{1}{\cos(A)}$, (secant of A), $\operatorname{cosec}(A)=\frac{1}{\sin(A)}$, (cosecant of A)

$\sin^2(A)+\cos^2(A)=1$ entered as $(\sin(A))^2+(\cos(A))^2$ MC

$\sin(-\theta)=-\sin(\theta)$ (an ODD function)

$\cos(-\theta)=+\cos(\theta)$ (an EVEN function)

14.6 Multiple / double angles

[K Singh (1) 213-222 (2) 223-234]

$\sin(A+B)=\sin(A)\cos(B)+\cos(A)\sin(B)$

$\sin(A-B)=\sin(A)\cos(B)-\cos(A)\sin(B)$

$\cos(A+B)=\cos(A)\cos(B)-\sin(A)\sin(B)$

$\cos(A-B)=\cos(A)\cos(B)+\sin(A)\sin(B)$

$\sin(2A)=2\sin(A)\cos(A)$

see 15.1.3

MC

$\cos(2A)=\cos^2(A)-\sin^2(A)$

$=2\cos^2(A)-1$

$\cos^2 A=\frac{1}{2}(\cos(2A)+1)$

$\cos(2A)=1-2\sin^2(A)$

$\sin^2(A)=\frac{1}{2}(1-\cos(2A))$

MC

$\tan(A+B)=\frac{(\tan(A)+\tan(B))}{(1-\tan(A)\tan(B))}$

$\tan(A-B)=\frac{(\tan(A)-\tan(B))}{(1+\tan(A)\tan(B))}$

$\tan(2A)=\frac{2\tan(A)}{(1-\tan^2(A))}$

14.6.1 Products to Sums

$$\sin(A)\cos(B)=\frac{1}{2}(\sin(A+B)+\sin(A-B))$$

$$\cos(A)\sin(B)=\frac{1}{2}(\sin(A+B)-\sin(A-B))$$

$$\cos(A)\cos(B)=\frac{1}{2}(\cos(A+B)+\cos(A-B))$$

$$\sin(A)\sin(B)=\frac{1}{2}(\cos(A-B)-\cos(A+B))$$

14.6.2 Sums to Products

$$\sin(A)+\sin(B)=2\sin\left(\frac{(A+B)}{2}\right)\cos\left(\frac{(A-B)}{2}\right)$$

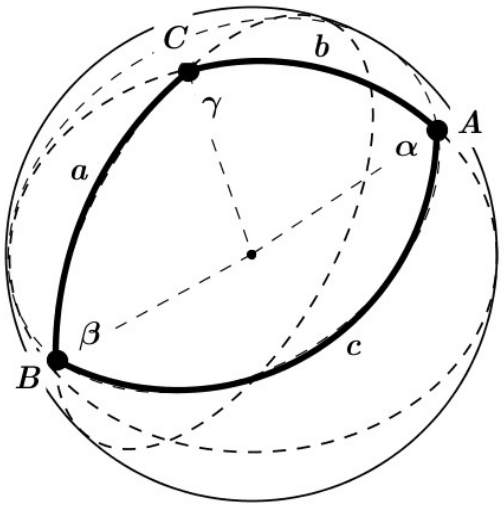
$$\sin(A)-\sin(B)=2\cos\left(\frac{(A+B)}{2}\right)\sin\left(\frac{(A-B)}{2}\right)$$

$$\cos(A)+\cos(B)=2\cos\left(\frac{(A+B)}{2}\right)\cos\left(\frac{(A-B)}{2}\right)$$

$$\cos(A)-\cos(B)=-2\sin\left(\frac{(A+B)}{2}\right)\sin\left(\frac{(A-B)}{2}\right)$$

14.7 Spherical Trigonometry

The sides of a spherical triangle are great circles (their diameter is that of the sphere). The sides, a,b,c are measured as angles from the centre of the sphere. In this case $a,b,c,\alpha,\beta,\gamma<180^o$, $0<a+b+c<360^o$, $180^o<\alpha+\beta+\gamma<540^o$



Sine Theorem

$$\frac{\sin\alpha}{\sin a}=\frac{\sin\beta}{\sin b}=\frac{\sin\gamma}{\sin c}$$

Side-cosine Theorem

$$\cos a=\cos b\cos c+\sin b\sin c\cos\alpha$$
$$\cos b=\cos c\cos a+\sin c\sin a\cos\beta$$
$$\cos c=\cos a\cos b+\sin a\sin b\cos\gamma$$

Angle-cosine Theorem

$$\cos\alpha=-\cos\beta\cos\gamma+\sin\beta\sin\gamma\cos a$$
$$\cos\beta=-\cos\gamma\cos\alpha+\sin\gamma\sin\alpha\cos b$$
$$\cos\gamma=-\cos\alpha\cos\beta+\sin\alpha\sin\beta\cos c$$

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15 Complex Numbers

[K Singh (1) 463-506 (2) 513-559]

Notation for Complex Numbers

BE - imaginary numbers

j	symbol representing $\sqrt{-1}$. (i used on most calculators) (defined as $j^2=-1$)
$a+jb$	a complex number in Cartesian (or Rectangular) form ($x+yi$ on a calculator). $a,b\in\mathbb{R}$, jb imaginary part.
z	a complex number $z=a+jb$ (or $x+yi$)
$r\angle\theta$	a complex number in polar form (A phasor representing $r\sin(\omega t+\theta)$)
$\overline{z}$	complex conjugate of the complex number If $z=a+jb$ then the complex conjugate $\overline{z}=a-jb$ or if $z=r\angle\theta$ then the complex conjugate $\overline{z}=r\angle-\theta$

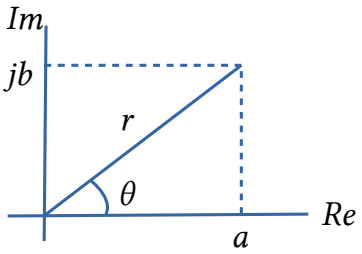
$$z=a+jb=r(\cos\theta+j\sin\theta)=r\angle\theta=re^{j\theta} \text{ where } j^2=-1$$

15.1.1 Complex Number Arithmetic

Modulus, (or magnitude) $r=|z|=\sqrt{a^2+b^2}$ see ,14.2 14.3

Argument, (or angle) $\theta=arg\,z=\tan^{-1}\left(\frac{b}{a}\right)$

BE - Complex arithmetic - better explained



Argand Diagram

Addition $(a+jb)+(c+jd)=(a+c)+j(b+d)$

Multiplication $(a+jb)(c+jd)$ (FOIL) Division $\frac{(a+jb)(c-jd)}{(c+jd)(c-jd)}$

Polar Multiplication $z_1z_2=r_1\angle\theta_1\times r_2\angle\theta_2=r_1r_2\angle(\theta_1+\theta_2)$ $((r\angle\theta)^2=r^2\angle2\theta)$

Polar Division $\frac{z_1}{z_2}=\frac{r_1\angle\theta_1}{r_2\angle\theta_2}=\frac{r_1}{r_2}\angle(\theta_1-\theta_2)$ $\left(\sqrt{r\angle\theta}=\sqrt{r}\angle\frac{\theta}{2}\right)$

See also: 16.2 Co-ordinate conversion MC

15.1.2 De Moivre's Theorem

$$(r\angle\theta)^n=r^n\angle(n\theta)=r^n(\cos(n\theta)+j\sin(n\theta))$$

http://www.justinmullins.com/home.htm

15.1.3 Phasors

A wave $y=r\sin(\omega t+\alpha)$ can be represented by a phasor (a rotating vector) $r\angle\alpha$

16 Vectors

Notation for Graphs and Vectors

[K Singh (1) 567-600 (2) 636-671]

(x,y) the co-ordinates of a point, where x is the distance from the y axis and y is the distance from the x axis

$\underline{v}$ a vector. Always underlined in written work. Bold $\boldsymbol{v}$ in type.

$\vec{AB}$ a vector, direction A to B.

$\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ or $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ a vector in Component form (Rectangular Form)

$a\underline{i}+b\underline{j}+c\underline{k}$ a vector in Cartesian form (Rectangular form) where $\underline{i}$ is a $a_1\underline{i}+a_2\underline{j}+a_3\underline{k}$ horizontal 1 unit vector and $\underline{j}$ is a vertical 1 unit vector.

$r\angle\theta$ a vector in polar form (where $r=|\underline{v}|=\sqrt{a^2+b^2}$, the modulus or magnitudeof vector $\underline{v}$. Use of $\|\underline{v}\|$ better

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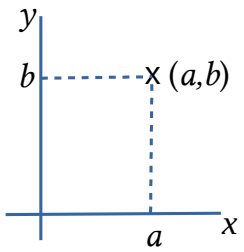
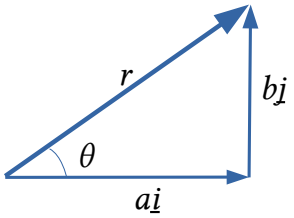
16.1.1 Vector Arithmetic





$r=\sqrt{a^2+b^2}$

$\theta=\tan^{-1}\left(\frac{b}{a}\right)$

$a=r\cos(\theta)$

$b=r\sin(\theta)$

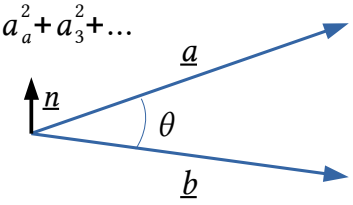
A point (a,b) A vector $\underline{v}=\begin{pmatrix} a \\ b \end{pmatrix}$ or $\underline{v}=r\angle\theta$ MC

Vector Addition $\begin{pmatrix} a \\ b \\ c \end{pmatrix}+\begin{pmatrix} d \\ e \\ f \end{pmatrix}=\begin{pmatrix} a+d \\ b+e \\ c+f \end{pmatrix}$ see also 16.2 and Geogebra vector addition

Scalar (Dot) Product $\underline{a}\cdot\underline{b}=|\underline{a}||\underline{b}|\cos(\theta)$ where $\underline{a}\cdot\underline{b}=a_1b_1+a_2b_2+a_3b_3\ldots$

and $|\underline{a}|=\sqrt{a_1^2+a_a^2+a_3^2+\ldots}$

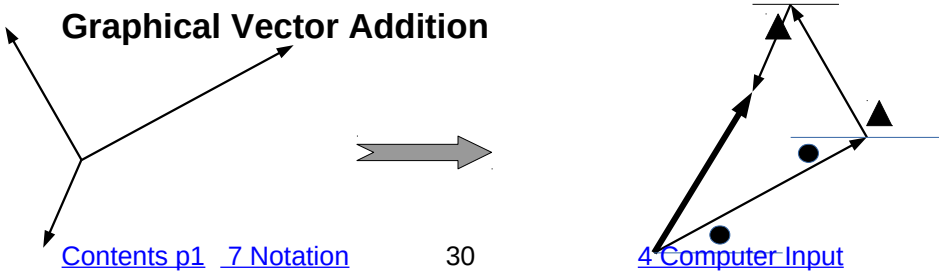
Note: $\underline{a}\cdot\underline{b}=\begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix}\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ see 12



Cross Product $\underline{a}\times\underline{b}=|\underline{a}||\underline{b}|\sin(\theta)\underline{n}$ where $\underline{n}$ is a perpendicular unit vector

and where $\underline{a}\times\underline{b}=det\begin{pmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}$ see section 12.1.1.5

16.1.2 Graphical Vector Addition



16.2 Co-ordinate Conversion using Scientific Calculators

R to P Rectangular to Polar $\begin{pmatrix} x \\ y \end{pmatrix}$ to $r\angle\theta$ ($(x+jy)$ to $r\angle\theta$)

P to R Polar to Rectangular $r\angle\theta$ to $\begin{pmatrix} x \\ y \end{pmatrix}$ ($r\angle\theta$ to $(x+jy)$)

Casio Natural Display and **Texet EV-S** **Edit keystrokes for your calculator**

R to P	SHIFT	Pol(	x	SHIFT	,	y	)	=	r out θ
P to R	SHIFT	Rec(	r	SHIFT	,	θ	)	=	x out y

Casio S-VPAM and new Texet **Edit keystrokes for your calculator**

R to P	SHIFT	Pol(	x	SHIFT	,	y	)	=	r out	RCL	tan	θ out
P to R	SHIFT	Rec(	r	SHIFT	,	θ	)	=	x out	RCL	tan	y out

Sharp WriteView

R to P	x	(x,y)	y	2ndF	$\rightarrow r\theta$	r,θ out
P to R	r	(x,y)	θ	2ndF	$\rightarrow x\,y$	x,y out

Sharp *ADVANCED* D.A.L. **Edit keystrokes for your calculator**

R to P	x	2ndF	,	y	2ndF	$\rightarrow r\theta$	r out	→	θ out
or				MATH	1	r out	2ndF	←·→	θ out
P to R	r	2ndF	,	θ	2ndF	$\rightarrow x\,y$	x out	→	y out
or				MATH	2	x out	2ndF	←·→	y out

Old Casio *fx* & VPAM

R to P	x	SHIFT	$R\rightarrow P$	y	=	r out	SHIFT	$X\rightarrow Y$	θ out
P to R	r	SHIFT	$P\rightarrow R$	θ	=	x out	SHIFT	$X\rightarrow Y$	y out

Texet - albert 2

R to P	x	INV	$x\Leftrightarrow y$	y	$R\rightarrow P$	r out	INV	$x\Leftrightarrow y$	θ out
P to R	r	INV	$x\Leftrightarrow y$	θ	$P\rightarrow R$	x out	INV	$x\Leftrightarrow y$	y out

Casio Graphics (1)

R to P	SHIFT	Pol(	x	SHIFT	,	y	)	EXE	r out	ALPHA	J	EXE	θ out
P to R	SHIFT	Rec(	r	SHIFT	,	θ	)	EXE	x out	ALPHA	J	EXE	y out

Casio Graphics (2)

R to P	FUNC	4	MATH	4	COORD	1	Pol(	x	,	y	)	EXE	r	ALPHA	J	EXE	θ
P to R	FUNC	4	MATH	4	COORD	1	Rec(	r	,	θ	)	EXE	x	ALPHA	J	EXE	y

Casio Graphics (7 series)

R to P	OPTN	▶	F2	▶	▶	Pol(	x	,	y	)	EXE	r, θ out
P to R	OPTN	▶	F2	▶	▶	Rec(	r	,	θ	)	EXE	x, y out

Old Texet and old Sharp and some £1 calculators

You must be in Complex Number mode.

					2ndF	CPLX				
R to P	x	a	y	b	2ndF	a	r out	b	θ out	
P to R	r	a	θ	b	2ndF	b	x out	b	y out	

Texas - 36X

R to P	x	$x \Leftrightarrow y$	y	3rd	$R \rightarrow P$	r out	$x \Leftrightarrow y$	θ out
P to R	r	$x \Leftrightarrow y$	θ	2nd	$P \rightarrow R$	x out	$x \Leftrightarrow y$	y out

Texas Graphics (TI 83)

R to P	2nd	Angle	$R \rightarrow Pr$ (	x, y	)	ENTER	r out
	2nd	Angle	$R \rightarrow P\theta$ (	x, y	)	ENTER	θ out
P to R	2nd	Angle	$P \rightarrow Rx$ (	r, θ	)	ENTER	x out
	2nd	Angle	$P \rightarrow Ry$ (	r, θ	)	ENTER	y out

Sharp Graphics

R to P	MATH	(D)CONV	(3) $xy \rightarrow r$ (	x	,	y	)	ENTER	r out
	MATH	(D)CONV	(4) $xy \rightarrow \theta$ (	x	,	y	)	ENTER	θ out
P to R	MATH	(D)CONV	(5) $r\theta \rightarrow x$ (	r	,	θ	)	ENTER	x out
	MATH	(D)CONV	(6) $r\theta \rightarrow y$ (	r	,	θ	)	ENTER	y out

Insert the keystrokes for your calculator here (if different from above)

R to P										
P to R										

Degrees to Radians $\div 180 \times \pi$ **Radians to degrees** $\div \pi \times 180$

---

17 Functions

17.1 Indices and Logarithms

[K Singh (1,2) 7-11, (1) 223-245 (2) 235-259]

17.1.1 Rules of Indices

notation 7.1.1 **MC**

1. $a^m \times a^n = a^{(m+n)}$

2. $\frac{a^m}{a^n} = a^{(m-n)}$

3. $(a^m)^n = a^{mn}$

4. $a^{\left(\frac{m}{n}\right)} = \sqrt[n]{a^m}$ $\qquad a^{\left(\frac{1}{n}\right)} = \sqrt[n]{a}$

5. $k a^{-n} = \frac{k}{a^n}$

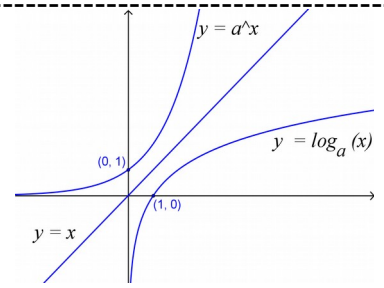
Also,

$a^0 = 1$ $\qquad \sqrt{x} = x^{\frac{1}{2}} = x^{0.5}$ and $\sqrt[3]{a} = \sqrt{a}$

$a^1 = a$ $\qquad \sqrt[n]{a} = b \Leftrightarrow b^n = a$

17.1.2 Definition of logarithms

If $N = a^n$ then $n = \log_a(N)$



17.1.3 Rules of logarithms

MC

1. $\log(A \times B) = \log(A) + \log(B)$

2. $\log\left(\frac{A}{B}\right) = \log(A) - \log(B)$

3. $\log(A^n) = n\log(A)$

4. $\log_a N = \frac{\log_b N}{\log_b a}$

---

ln button	log button
$\exp(x) = e^{(x)}$ $\qquad \log_e(x) = \ln(x)$	$\log_{10}(x) = \lg(x)$

17.2 Infinite Series and Hyberbolic Functions

[K Singh (1) pp 246-346, 338-346 (2) 259-270, 358-369]

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \ldots \text{ for } |x| < \infty$$

BE exponential functions better explained

$$\sin x = \frac{e^{jx} - e^{-jx}}{j2} \left(= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \right) \quad \text{for } |x| < \infty$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2} \left(= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots \right) \quad \text{for } |x| < \infty$$

$$\ln x = \frac{x-1}{1} - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \ldots \quad \text{for } 0 < x \leq 2$$

BE- demystifying the natural logarithm

Hyperbolic Functions

- definitions [K Singh (1) 246-247 (2) 259-260]

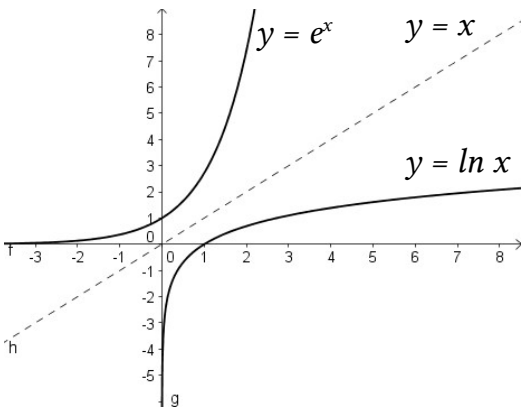
MC

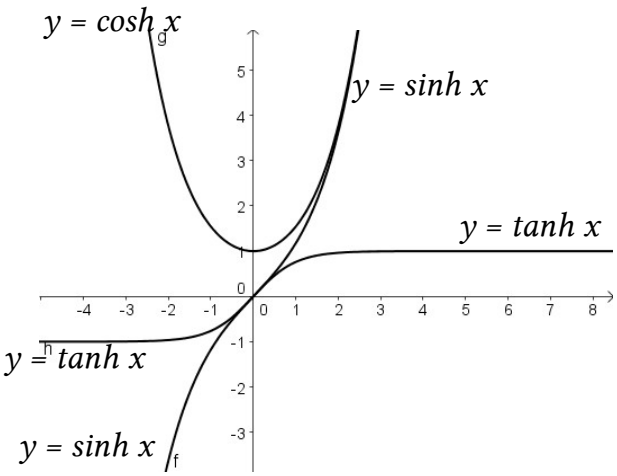
pronunciation

$$\sinh x = \frac{e^x - e^{-x}}{2} \left(= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots \right) \quad \text{“shine x”}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \left(= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots \right) \quad \text{“cosh x”}$$

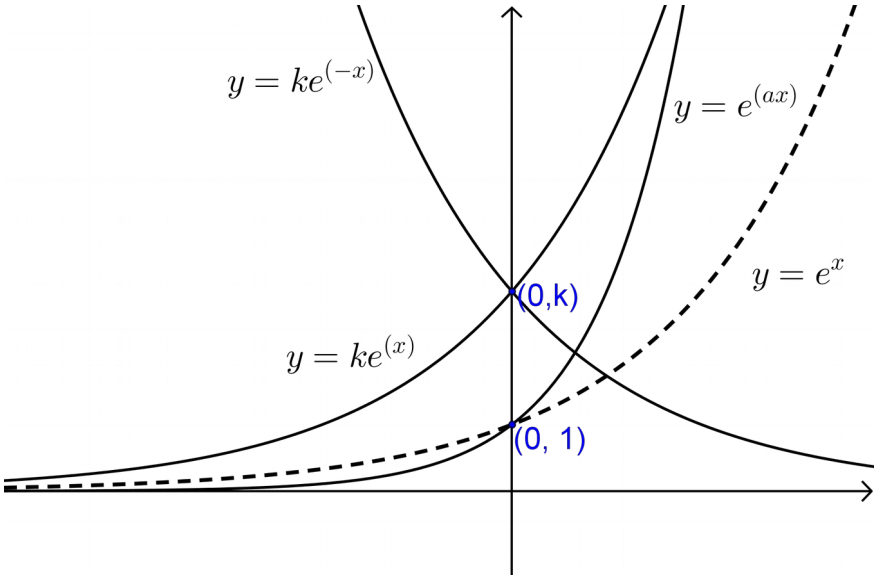
$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad \text{“thaan x”}$$

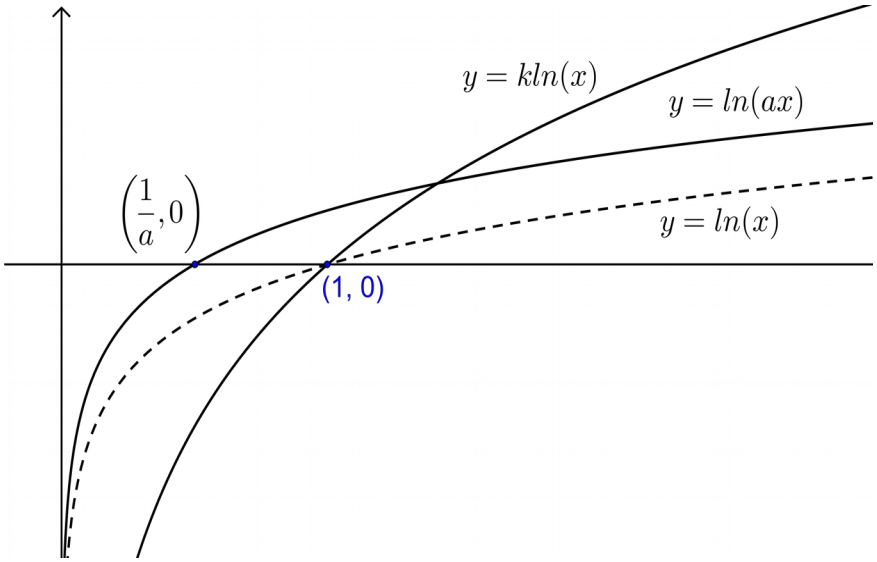


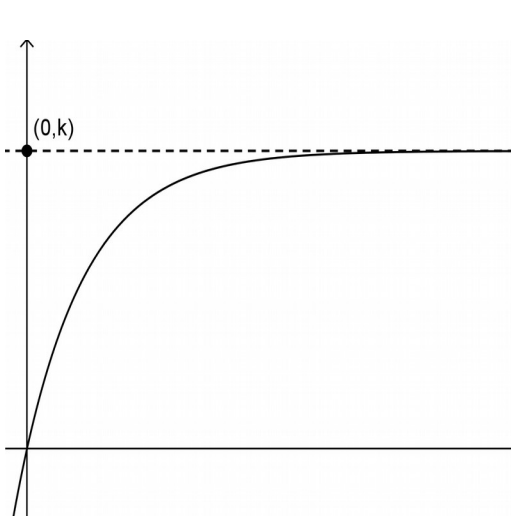


$k e^{ax}$ slider $k \ln(ax)$ slider

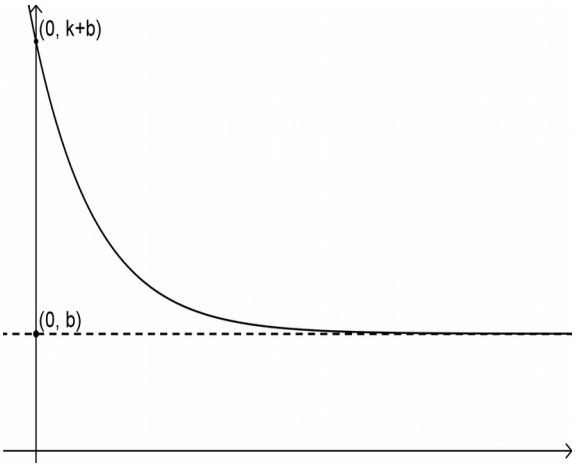
17.3 Exponential and Logarithmic Graphs







$y=k\left(1-e^{(-\alpha t)}\right)$



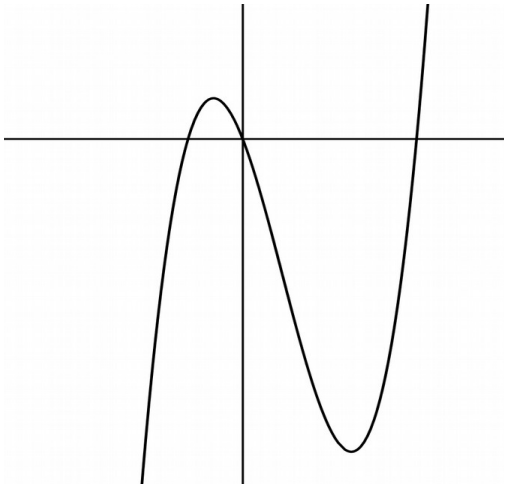
$y=k\,e^{(-\alpha t)}+b$

Horizontal asymptotes at

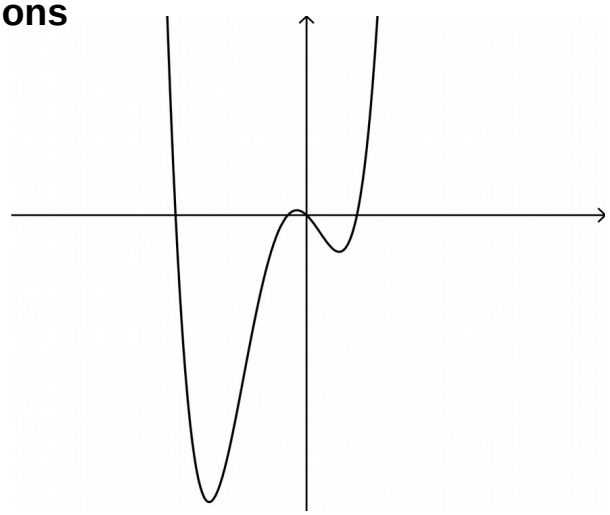
$y=k$

$y=b$

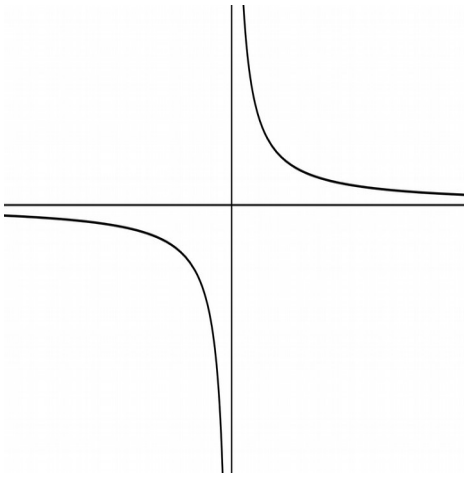
17.4 Graphs of Common Functions



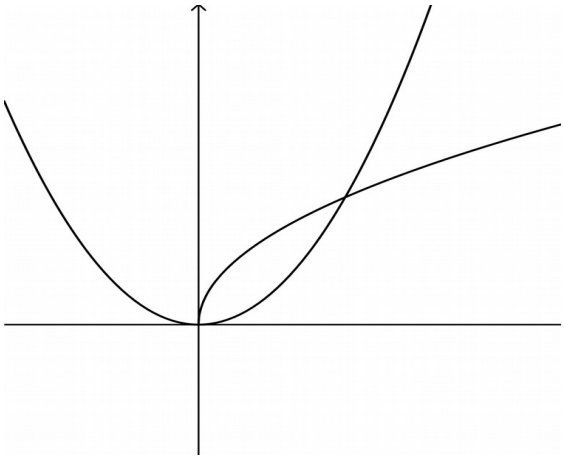
$y=ax^3+bx^2+cx+d$



$y=ax^4+bx^3+cx^2+dx+f$

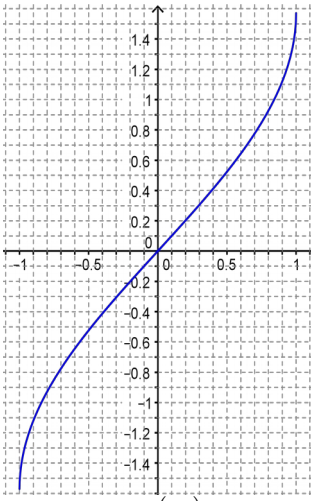


$y=\frac{a}{x}+b$

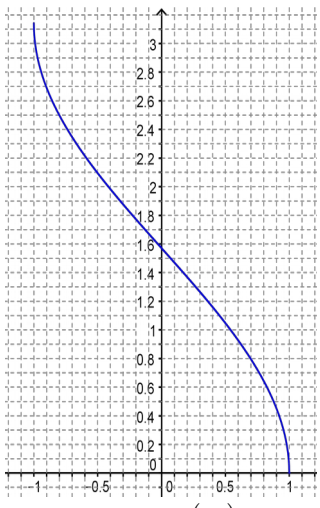


$y=x^2$ and $y=+\sqrt{x}$

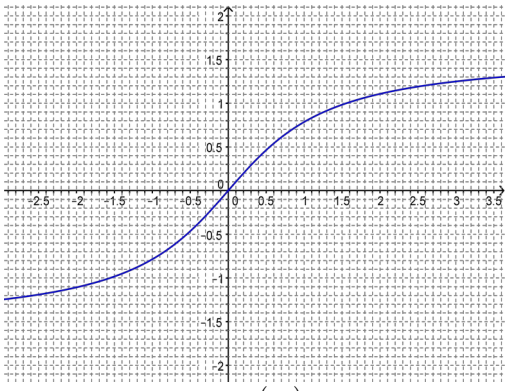
Vertical asymptote at $x=0$



$y=\text{asin}(x)$
$y=\sin^{-1}(x)$



$y=\text{acos}(x)$
$y=\cos^{-1}(x)$



$y=\text{atan}(x)$
$y=\tan^{-1}(x)$

See also 14.1.1

18 Calculus

18.1.1 Notation for Calculus

see also section 7

Differentiation

$\frac{dy}{dx}$	the first derivative of y where y is a function of x (Leibniz) Rate of change of y with respect to x. Also see 9.
$f'(x)$	the first derivative of $f(x)$. (as above). (Lagrange)
$\dot{v}$	the first derivative of v w.r.t. time. (Newtonian mechanics)
$D(u)$	the first derivative of u
$\frac{\Delta y}{\Delta x}$	Change in y over change in x.
$\frac{d^2y}{dx^2}$	the second derivative of y w.r.t x. The $\frac{dy}{dx}$ of $\frac{dy}{dx}$
$f''(x)$	the second derivative of $f(x)$. ($f^2(x)$ is also used)
$\ddot{v}$	the second derivative of v w.r.t. time. (Newtonian mechanics)
$\frac{\partial z}{\partial x}$	the partial derivative of z w.r.t. x. (∂ "partial d")
δx	a small change (increment) in x . (δ "delta")

Integration

$\int dx$	the integral sign (Summa) (with respect to x)
$\int f(x)dx$	the indefinite integral of $f(x)$ (if $\frac{dy}{dx}=f(x)$ then $y=\int f(x)dx$)
$\int_a^b f(x)dx$	the definite integral of $f(x)$ from $x=a$ to $x=b$ the area under $f(x)$ between $x=a$ and $x=b$
$F(x)$	the primitive of $f(x)$ ($\int f(x)dx$ without the c)
$L[f(t)]$	the Laplace operator (with parameter s)

18.2 Differential Calculus - Derivatives

[K Singh (1) pp 258 - 358 (2) 271-398]

y or $f(x)$	$\frac{dy}{dx}$ or $f'(x)$
x^n	$n x^{n-1}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
e^x	e^x
$\ln x$	$\frac{1}{x}$

k	0
$k x^n$	$k n x^{n-1}$
$\sin(a x)$	$a \cos(a x)$
$\cos(a x)$	$-a \sin(a x)$
$e^{(a x)}$	$a e^{(a x)}$
$\ln(a x)$	$\frac{a}{a x} = \frac{1}{x}$

$k(a x+b)^n$	$k n a(a x+b)^{n-1}$
$k \sin(a x+b)$	$k a \cos(a x+b)$
$k \cos(a x+b)$	$-k a \sin(a x+b)$
$k \tan(a x+b)$	$k a \sec^2(a x+b) = \frac{k a}{\cos^2(a x+b)}$
$k e^{(ax+b)}$	$k a e^{(ax+b)}$
$k \ln(a x+b)$	$\frac{k a}{(a x+b)}$

See 9 , MC

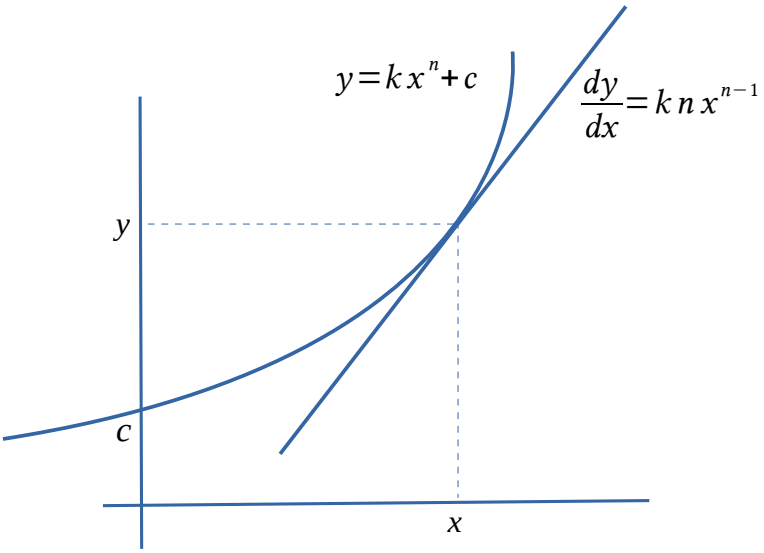
The basics – all you need if you know the Chain Rule.

e^x **gradient slider**

Further Standard Derivatives

y or $f(x)$	$\frac{dy}{dx}$ or $f'(x)$	
$\ln[f(x)]$	$\frac{f'(x)}{f(x)}$	
$\sin^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{a^2-x^2}}, \quad x^2<a^2$	
$\cos^{-1}\left(\frac{x}{a}\right)$	$\frac{-1}{\sqrt{a^2-x^2}}, \quad x^2<a^2$	
$\tan^{-1}\left(\frac{x}{a}\right)$	$\frac{a}{a^2+x^2}$	
$\sinh(ax+b)$	$a\cosh(ax+b)$	
$\cosh(ax+b)$	$a\sinh(ax+b)$	see 17.2
$\tanh(ax+b)$	$a\,\mathrm{sech}^2(ax+b)$	
$\sinh^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{x^2+a^2}}$	
$\cosh^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{x^2-a^2}}, \quad x^2>a^2$	
$\tanh^{-1}\left(\frac{x}{a}\right)$	$\frac{a}{a^2-x^2}, \quad x^2<a^2$	

Differentiation as a gradient function (tangent to a curve) (rate of change).

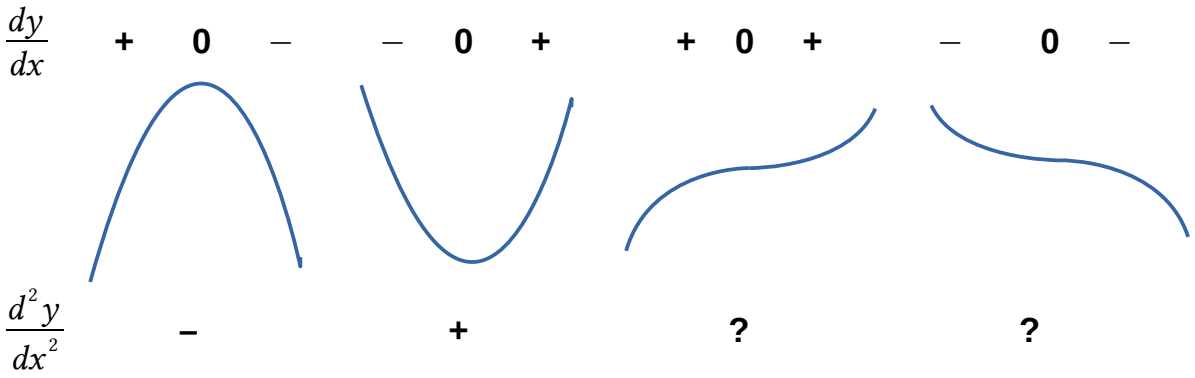


18.2.1 Maxima and Minima

(Stationary Points) [K Singh (1) 308-335 (2) 327-354]

If $y=f(x)$ then at any turning point or stationary point $\frac{dy}{dx}=f'(x)=0$

Determine the nature (max, min or saddle) of the turning points by evaluating gradients locally (i.e. close to turning point**).** MC



18.2.2 Differentiation Rules

[K Singh (2) 274–285 (2) 286-302]

For D read *differentiate* $D[k f(x)]=k f'(x)$, k a constant

Function of a function rule $D[f(g(x))]=f'(g(x))\times g'(x)$

(Chain Rule) $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$ **MC**

If u and v are functions of x then:

Addition Rule $D(u+v)=\frac{du}{dx}+\frac{dv}{dx}=u'+v'$

Product Rule $D(uv)=v\frac{du}{dx}+u\frac{dv}{dx}=v\,u'+u\,v'$ MC

Quotient Rule $D\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}=\frac{vu'-uv'}{v^2}$ **MC**

18.2.3 Formula for the Newton-Raphson Iterative Process

[K Singh (1) pp 352 - 356 (2) 389-398]

Set $f(x)=0$ with guess value x_0 (from graph) see 9

Test for Convergence $\left|\frac{f(x_0)f''(x_0)}{[f'(x_0)]^2}\right|<1$ see 7 - modulus

x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$

(where $f'(x_n)\neq 0$)

$f(x)=0$ when $x_{n+1}=x_n$ to the precision required.

http://archives.math.utk.edu/visual.calculus/3/newton.5/1.html

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18.2.4 Partial Differentiation

[K Singh (1) 695-725, (2) 772-805]

If $z=f(x,y)$ then a small change in x, named δx (delta x) and a small change in y, named δy etc. will cause a small change in z, named δz such that $\delta z\simeq\frac{\partial z}{\partial x}\delta x+\frac{\partial z}{\partial y}\delta y+\ldots$ where $\frac{\partial z}{\partial x}$ is the partial derivative of z w.r.t. x and $\frac{\partial z}{\partial y}$ is the partial derivative of z w.r.t y. see 7

18.2.5 Implicit Differentiation

[K Singh (1) 298-306 (2) 315-325]

If $z=f(x,y)$ then $\frac{dy}{dx}=\frac{\left(\frac{\partial z}{\partial x}\right)}{\left(\frac{\partial z}{\partial y}\right)}$ Also $\frac{dy}{dx}=\frac{1}{\left(\frac{dx}{dy}\right)}$

18.2.6 Parametric Differentiation

[K Singh (1) 291-296 (2) 308-315]

If $x=f(t)$ and $y=g(t)$

$\frac{dx}{dt}=f'(t)$ and $\frac{dy}{dt}=g'(t)$

$\frac{dy}{dx}=\frac{g'(t)}{f'(t)}$ or $\frac{dy}{dx}=\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$ $\left(f'(t),\frac{dx}{dt}\neq 0\right)$ MC

18.3 Integral Calculus - Integrals

[K Singh (1) 359-462 (2) 399-512] $\int$

$\frac{dy}{dx}$ or $f(x)$	y or $\int f(x)dx$ or $F(x)$ + **c**	
x^n	$\frac{x^{n+1}}{n+1}$	$n \neq -1$
$\sin x$	$-\cos x$	
$\cos x$	$\sin x$	
e^x	e^x	
$\frac{1}{x} = x^{-1}$	$\ln x$	when $n=-1$

The basics – all you need if you know the Substitution Rule.

k	kx	
kx^n	$\frac{kx^{n+1}}{n+1}$	$n \neq -1$
$\sin(ax)$	$\frac{-\cos(ax)}{a}$	
$\cos(ax)$	$\frac{\sin(ax)}{a}$	
$e^{(ax)}$	$\frac{e^{(ax)}}{a}$	
$\frac{k}{x} = kx^{-1}$	$k \ln x$	when $n=-1$

$k(ax+b)^n$	$\frac{k(ax+b)^{n+1}}{(n+1)a}$	$n \neq -1$
$k\sin(ax+b)$	$\frac{-k\cos(ax+b)}{a}$	
$k\cos(ax+b)$	$\frac{k\sin(ax+b)}{a}$	
$k\sec^2(ax+b)$	$\frac{k\tan(ax+b)}{a}$	
$ke^{(ax+b)}$	$\frac{ke^{(ax+b)}}{a}$	
$\frac{k}{(ax+b)}$	$\frac{k\ln(ax+b)}{a}$	$n=-1$

Further Standard Integrals

$\frac{dy}{dx}$ or $f(x)$	y or $\int f(x)\,dx$ or $F(x)$ + **c**	
$\frac{1}{\sqrt{a^2-x^2}}$, $x^2<a^2$	$\sin^{-1}\left(\frac{x}{a}\right)$	
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	
$\sinh(ax+b)$	$\frac{1}{a}\cosh(ax+b)$	
$\cosh(ax+b)$	$\frac{1}{a}\sinh(ax+b)$	see 17.2
$\text{sech}^2(ax+b)$	$\frac{1}{a}\tanh(ax+b)$	
$\frac{1}{\sqrt{x^2+a^2}}$, $x^2>a^2$	$\sinh^{-1}\left(\frac{x}{a}\right)$ or $\ln(x+\sqrt{x^2+a^2})$	
$\frac{1}{\sqrt{x^2-a^2}}$, $x^2>a^2$	$\cosh^{-1}\left(\frac{x}{a}\right)$ or $\ln(x+\sqrt{x^2-a^2})$	
$\frac{1}{a^2-x^2}$, $x^2<a^2$	$\frac{(\ln(a+x)-\ln(a-x))}{2a}$	
$\frac{1}{x^2-a^2}$, $x^2>a^2$	$\frac{(\ln(x-a)-\ln(x+a))}{2a}$	

- -

$\left(\frac{\frac{dy}{dx}}{y}\right)$ $\frac{f'(x)}{f(x)}$ $\qquad$ $\ln(f(x))$ $\quad$ $(\ln(y))$

Addition Rule $\quad \int f(x)+g(x)\,dx=\int f(x)\,dx+\int g(x)\,dx$

18.3.1 Integration by Substitution

[K Singh (1) 368 (2) 414]

$k\int g'(x)f(g(x))\,dx$ $\qquad$ MC

$k\int f(u)\,du$ where $u=g(x)$ then $du=g'(x)\,dx$ and $dx=\frac{du}{g'(x)}$

Note change of limits $\int_{x=a}^{x=b} g'(x)f(g(x))\,dx$ to $\int_{u\text{ when }x=a}^{u\text{ when }x=b} f(u)\,du$

du is a function of u or $du\in\mathbb{R}$ $\qquad$ Notes and exercises

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18.3.2 Integration by Parts

[K Singh (1) 388-395 (2) 432-440]

$\int u\,dv=uv-\int v\,du$ $\qquad$ see 18.5 $\quad$ **MC**

Notes and exercises

18.3.3 Solution of Differential Equations (Indefinite Integration)

$$\frac{dy}{dx} = f(x)$$

$$dy = f(x)\,dx$$

$$\int 1\,dy = \int f(x)\,dx$$

$$y = \int f(x)\,dx$$ MC

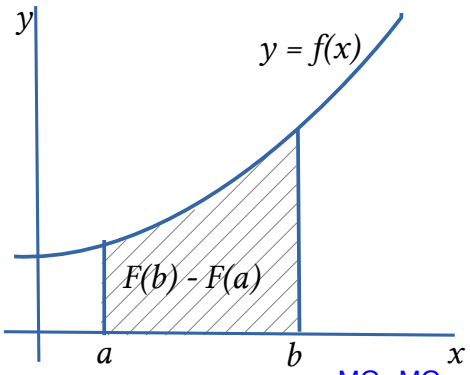
$$y = F(x) + c$$ (Boundary conditions or initial values required to find c)

18.3.4 Area under a Curve - Definite Integration [K Singh (1) 442 (2) 489]

$$\int_a^b f(x)\,dx$$

$$= \left[F(x) + c\right]_a^b$$

$$= \left(F(b) + c\right) - \left(F(a) + c\right)$$



MC, MC

Procedure

Plot between limits - a and b

Check for roots $\left(R_1, R_2, \ldots, R_n\right)$ and evaluate

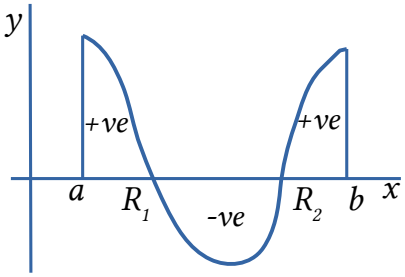
See Newton Raphson 18.2.3

Integrate between left limit, a, and R_1

then between R_1 and R_2 and so on to

last root R_n and right limit b

Add moduli of areas. (areas all +ve)

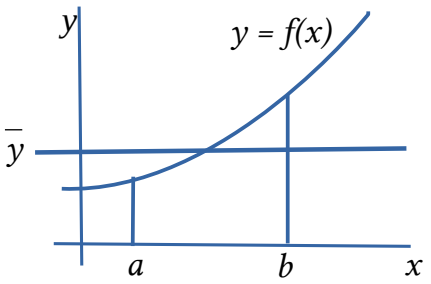


18.3.5 Mean Value

[K Singh (1) p 445 (2) 492]

If $y = f(x)$ then $\bar{y}$,
the mean (or average) value of y
over the interval $x = a$ to $x = b$ is

$$\bar{y} = \frac{\int_a^b y\,dx}{(b-a)}$$



18.3.6 Root Mean Square (RMS)

$$y_{rms} = \sqrt{\frac{1}{b-a}\int_a^b y^2\,dx}$$ where $y = f(x)$

18.3.7 Volume of Revolution around the x axis

[J Bird 207-208] MC

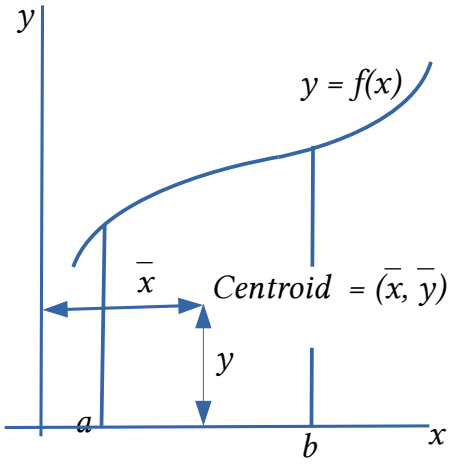
$$V = \pi \int_a^b y^2\,dx$$ where $y = f(x)$

18.3.8 Centroid

[J Bird 208 - 210]

The centroid of the area of a lamina bounded by a curve $y=f(x)$ and limits $x=a$ and $x=b$ has co-ordinates $(\bar{x},\bar{y})$.

$$\bar{x}=\frac{\int_a^b x\,y\,dx}{\int_a^b y\,dx} \quad \text{and} \quad \bar{y}=\frac{\frac{1}{2}\int_a^b y^2\,dx}{\int_a^b y\,dx}$$



18.3.9 Partial Fractions

[K Singh (1) 396-410 (2) 440-455]

$$\frac{f(x)}{(x+a)(x+b)}\equiv\frac{A}{(x+a)}+\frac{B}{(x+b)}$$ see 8

$$\frac{f(x)}{(x+a)^2(x+b)}\equiv\frac{A}{(x+a)}+\frac{B}{(x+a)^2}+\frac{C}{(x+b)}$$

$$\frac{f(x)}{(x^2+a)(x+b)}\equiv\frac{Ax}{(x^2+a)}+\frac{B}{(x^2+a)}+\frac{C}{(x+b)}$$

MC

18.3.9.1 Completion of the Square

If $y=x^2+bx+d$ then $y=\left(x+\frac{b}{2}\right)^2+c$ See Quadratics

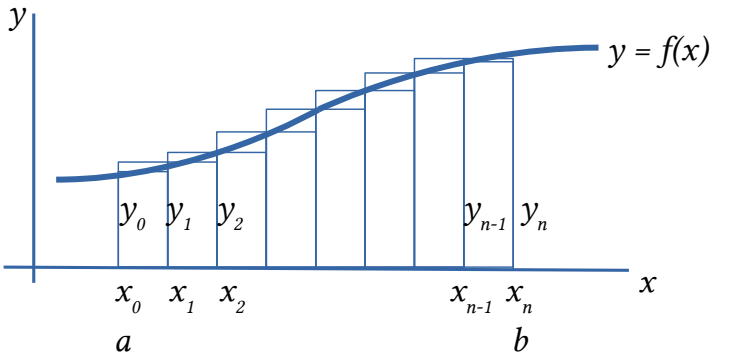
$$y=x^2+bx+\frac{b^2}{4}+c \qquad d=\frac{b^2}{4}+c \text{ Find } c.$$

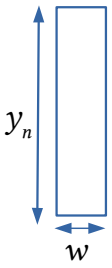
18.3.10 Approximation of Definite Integrals

[K Singh (1) 434 (2) 481]

18.3.10.1 Simpson's Rule

$$w=\frac{b-a}{n}$$





$$\int_a^b f(x)\,dx \approx Area \approx \frac{w}{3}\left(y_0+4\,y_1+2\,y_2+\ldots+2\,y_{n-1}+4\,y_{n-1}+y_n\right) \qquad (n \text{ is even})$$

$$\int_a^b f(x)\,dx \approx \frac{w}{3}\left[\text{first}+\text{last}+4\left(\sum \text{odds}\right)+2\left(\sum \text{evens}\right)\right]$$

$y_1, y_3, \ldots$ odd,
$y_2, y_4, \ldots$ even

n	x_n	y_n	Multiplier m	Product $m\,y_n$
0	a	y_0	1	$1\times y_0$
1	$a+w$	y_1	4	$4\times y_1$
2	$a+2\,w$	y_2	2	$2\times y_2$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
$n-2$	.	y_{n-2}	2	$2\times y_{n-2}$
$n-1$	.	y_{n-1}	4	$4\times y_{n-1}$
n	b	y_n	1	$1\times y_n$
			Sum =	
			$\times w$ =	
			$\div 3$ =	

18.3.10.2 Trapezium Method

$$\int_a^b f(x)\,dx \approx \frac{w}{2}\left(y_0+2\,y_1+2\,y_2+\ldots..+2\,y_{n-1}+y_n\right)$$

18.4 Differential Equations

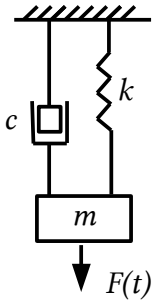
MC

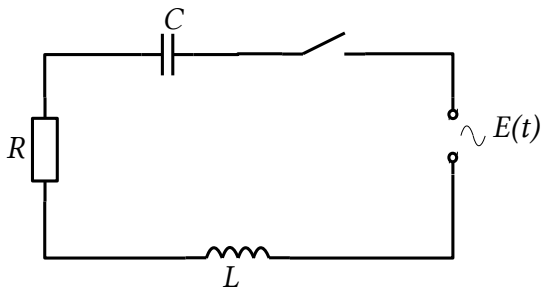
18.4.1.1 General Mathematical Model for Differential Equations

Mechanical Quantity	Electrical Quantity	voltage drop
mass (m)	inductance (L)	$v = L\frac{dI}{dt}$
position (y)	electric charge (Q)	
velocity $\left(v = \frac{dy}{dt}\right)$	electric current $\left(I = \frac{dQ}{dt}\right)$	
damping constant (c) friction coefficient (μ)	resistance (R)	$v = I\,R$
spring constant (k)	1/capacitance $\left(\frac{1}{C}\right)$	$\frac{dv}{dt} = \frac{I}{C}$
external force $(F(t))$	electromotive force (E)	

$$m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + k\,y = F(t) \qquad L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E(t)$$

$$L\frac{dI}{dt} + R\,I + \frac{Q}{C} = E(t) \qquad L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{I}{C} = \frac{d}{dt}(E(t))$$





18.4.2 Solution of Differential Equations

1. Indefinite Integration

$$\frac{dy}{dx} = f(x)$$
$$dy = f(x)\,dx$$
$$\int 1\,dy = \int f(x)\,dx$$
$$y = F(x) + c$$

MC

2. Extended to Separation of Variables $\frac{dy}{dx} = f(x, y)$ MC MC

18.4.2.1 Solution of Differential Equations using wxMaxima

$\frac{dy}{dx}$ typed as ‘diff(y,x) *note the apostrophe ‘ before* diff

$\frac{d^2y}{dx^2}$ typed as ‘diff(y,x,2)

Equations; Solve ODE. Equations; Initial value problem (1) or (2).

18.4.3 Table of Laplace Transforms

$\mathcal{L}[f(t)]$ is defined by $\int_0^\infty f(t)e^{-st}dt$ and is written as $F(s)$ [J Bird 582 – 604]

	$f(t)$	$\mathcal{L}[f(t)]$
1	1	$\frac{1}{s}$ $(\mathcal{L}[0]=0)$
1	k	$\frac{k}{s}$
2	t	$\frac{1}{s^2}$
3	t^n	$\frac{n!}{s^{n+1}}$
4	e^{-at}	$\frac{1}{s+a}$
5	$1-e^{-at}$	$\frac{a}{s(s+a)}$
6	te^{-at}	$\frac{1}{(s+a)^2}$
7	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
8	$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
9	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
10	$1-\cos(\omega t)$	$\frac{\omega^2}{s(s^2+\omega^2)}$
11	$\omega t\sin(\omega t)$	$\frac{2\omega^2 s}{(s^2+\omega^2)^2}$
12	$\sin(\omega t)-\omega t\cos(\omega t)$	$\frac{2\omega^3}{(s^2+\omega^2)^2}$
13	$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$ see 10
14	$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
15	$e^{-at}\left(\cos(\omega t)-\frac{a}{\omega}\sin(\omega t)\right)$	$\frac{s}{(s+a)^2+\omega^2}$
16	$\sin(\omega t+\phi)$	$\frac{s\sin\phi+\omega\cos\phi}{s^2+\omega^2}$
17	$e^{-at}+\frac{a}{\omega}\sin(\omega t)-\cos(\omega t)$	$\frac{a^2+\omega^2}{(s+a)(s^2+\omega^2)}$
18	$\sinh(\beta t)$	$\frac{\beta}{s^2-\beta^2}$
19	$\cosh(\beta t)$	$\frac{s}{s^2-\beta^2}$
20	$e^{-at}\sinh(\beta t)$	$\frac{\beta}{(s+a)^2-\beta^2}$
21	$e^{-at}\cosh(\beta t)$	$\frac{s+a}{(s+a)^2-\beta^2}$

18.4.3.1 Laplace Transforms of Differentials

First order differential equation:

$\mathcal{L}\left[\frac{dy}{dt}\right]=s\,\mathcal{L}[y]-y(0)$ where $y(0)$ is the value of y at $t=0$

Second order differential equation:

$\mathcal{L}\left[\frac{d^2y}{dt^2}\right]=s^2\mathcal{L}[y]-s\,y(0)-y'(0)$ where $y'(0)$ is the value of $\frac{dy}{dt}$ at $t=0$

MC Efunda Calculator Efunda - Laplace

18.4.4 Differential Equations - Auxiliary Equation Method

[K Singh (1) 656-670 (2) 730-746]
[J Bird 553-538]

Second-order homogeneous equations of the form $a\frac{d^2y}{dt^2}+b\frac{dy}{dt}+cy=0$ have auxiliary (or characteristic) equations of the form $a\lambda^2+b\lambda+c=0$ 10

or $ax^2+bx+c=0$

Solve the auxiliary equation for λ_1 and λ_2 (the roots) and if

$\lambda_1,\lambda_2\in\mathbb{R}$	$y(t)=Ae^{\lambda_1 t}+Be^{\lambda_2 t}$	or
$\lambda_1,\lambda_2\in\mathbb{R}$ and $\lambda_1=\lambda_2$	$y(t)=(A+Bt)e^{\lambda t}$	or
$\lambda_1=\alpha+j\beta$ and $\lambda_2=\alpha-j\beta$	$y(t)=e^{\alpha t}(A\cos(\beta t)+B\sin(\beta t))$	

A and B are arbitrary constants with values obtained from the boundary conditions or initial conditions. Differentiation will be required.

Second-order non-homogeneous equations of the form $a\frac{d^2y}{dt^2}+b\frac{dy}{dt}+cy=f(t)$ cannot be solved in the same way. [K Singh (1) 670-693 (2) 746-771], [J Bird 539-581]

MC

See also 12.1.1.3

18.4.5 Approximate numerical solution of differential equations

[K Singh (1) 630-655 (2) 703-729] and 26.1 [K Singh (1) 601-693 (2) 672-771]

18.4.5.1 Eulers' method

$y_1 = y_0 + h(y')_0$ See 9 Range $x = a(h)b$

where $y'(0) = \frac{dy}{dx} = f(x,y)$, h is the step size, a $(= x_0)$ and b are limits

and (x_0, y_0) is the boundary. Spreadsheet Method below

x_0	y_0	$(y')_0$ $\left(\frac{dy}{dx}\right)$	$y_1 = y_0 + h(y')_0$

Plot the graph of y against x from values in first 2 columns.

18.4.5.2 Euler's Method - Spreadsheet Method

$\frac{dy}{dx} = f(x,y)$, step size h and boundary (x_0, y_0)

	A	B	C	D
1		step size	h	
2	x	y	dy/dx	y1=y0+hX(dy/dx)
3	x_0	y_0	$f(x_0, y_0)$	=B3+$C$1*C3
4	=A3+$C$1	=D3	Fill down from C3 & D3 to C4 & D4	
5	Fill down all columns from here			

Format cells to give 5 dp
$ signs required to retain absolute column and row
Input values in C1 and A3 and B3 and the formula in C3
Plot columns A and B as a scatter graph with column A as x axis. 9.1.1

Thanks to Andrew Henderson

18.4.5.3 Runge-Kutta 4th Order

x_0	y_0	Intermediate gradient values	$y_1 = y_0 + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4)$
		$K_1 = f(x_0, y_0) = (y')_0$	
		$K_2 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}K_1\right)$	
		$K_3 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}K_2\right)$	
		$K_4 = f(x_0 + h, y_0 + hK_3)$	

18.4.5.4 Runge-Kutta - wxMaxima

$rk(f(x,y), y, y_0, [x, x_0, x_{end}, h])$ where $\frac{dy}{dx} = f(x,y)$

To plot result: wxplot2d([discrete,%o#],[style,points])
you can replace points *with* line. *%o# is a previous output line*.

18.5 Fourier Series.

[J Bird pp 611 - 657] and next page and 4 and 4.1

For period T, the smallest period of $f(t)$. (determine from a graph)

Fundamental angular frequency $\omega = \frac{2\pi}{T}$

$$f(t) = a_0 + a_1\cos(\omega t) + a_2\cos(2\omega t) + a_3\cos(3\omega t) + \ldots$$
$$+ b_1\sin(\omega t) + b_2\sin(2\omega t) + b_3\sin(3\omega t) + \ldots$$

a_n, b_n constants

or $f(t) = a_0 + \sum_{n=1}^{\infty}(a_n\cos(n\omega t) + b_n\sin(n\omega t))$

where $a_0 = \frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} f(t)\,dt$ mean value of $f(t)$ over period T see 18.3.2

$$a_n = \frac{2}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} f(t)\cos(n\omega t)\,dt \qquad n = 1, 2, 3\ldots$$

$$b_n = \frac{2}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} f(t)\sin(n\omega t)\,dt \qquad n = 1, 2, 3\ldots$$

Alternatively written as:

$f(t)$ = constant + first harmonic + second harmonic + ... see 14.4.2

$f(t) = a_0 + c_1\sin(\omega t + \alpha_1) + c_2\sin(2\omega t + \alpha_2) + \ldots + c_n\sin(n\omega t + \alpha_n)$ a_0 constant

$c_n = \sqrt{(a_n^2 + b_n^2)}$ and $\alpha_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$ Be careful of Quadrants!

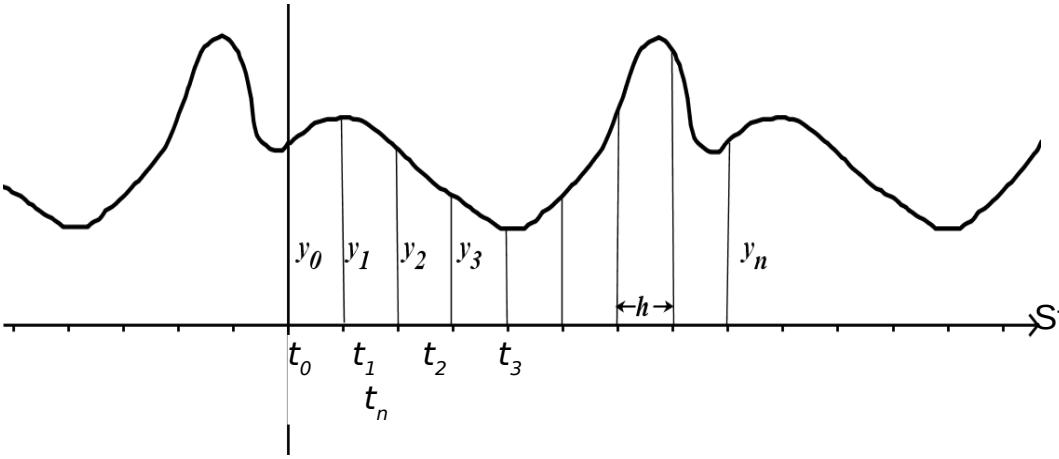
Or, as phasor addition $b_n + j\,a_n = c_n \angle \alpha_n$ or $\begin{pmatrix} b_n \\ a_n \end{pmatrix} = c_n \angle \alpha_n$ See 15, 16.2

Also $c_n \angle \alpha_n = c_n e^{j\alpha}$

See Fourier series applet http://www.falstad.com/fourier/index.html

- -

18.5.1 Fourier Series: A numerical method



$y_0 = y_n$

$\omega = \frac{2\pi}{T}$

Step size $h = \frac{T}{n}$

$t_{i+1} = t_i + h$

The table below will generate the first three harmonics

t_i	y_i	a_1	b_1	a_2	b_2	a_3	b_3
t_1	y_1	$y_1\cos(\omega t_1)$	$y_1\sin(\omega t_1)$	$y_1\cos(2\omega t_1)$	$y_1\sin(2\omega t_1)$	$y_1\cos(3\omega t_1)$	$y_1\sin(3\omega t_1)$
t_2	y_2	$y_2\cos(\omega t_2)$	$y_2\sin(\omega t_2)$	$y_2\cos(2\omega t_2)$	$y_2\sin(2\omega t_2)$	$y_2\cos(3\omega t_2)$	$y_2\sin(3\omega t_2)$
t_3	y_3	$y_3\cos(\omega t_3)$	$y_3\sin(\omega t_3)$	$y_3\cos(2\omega t_3)$	$y_3\sin(2\omega t_3)$	$y_3\cos(3\omega t_3)$	$y_3\sin(3\omega t_3)$
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
t_n	y_n	$y_n\cos(\omega t_n)$	$y_n\sin(\omega t_n)$	$y_n\cos(2\omega t_n)$	$y_n\sin(2\omega t_n)$	$y_n\cos(3\omega t_n)$	$y_n\sin(3\omega t_n)$
	$\sum y$	$\sum y\cos(\omega t)$	$\sum y\sin(\omega t)$	$\sum y\cos(2\omega t)$	$\sum y\sin(2\omega t)$	$\sum y\cos(3\omega t)$	$\sum y\sin(3\omega t)$
	$a_0 = \frac{\sum y}{n}$	$a_1 = \frac{2\sum y\cos(\omega t)}{n}$	$b_1 = \frac{2\sum y\sin(\omega t)}{n}$	$a_2 = \frac{2\sum y\cos(2\omega t)}{n}$	$b_2 = \frac{2\sum y\sin(2\omega t)}{n}$	$a_3 = \frac{2\sum y\cos(3\omega t)}{n}$	$b_3 = \frac{2\sum y\sin(3\omega t)}{n}$

Spreadsheet Method below
Calculate ω to at least 5 s.f. (or exact value) and use as w in the table below

	A	B	C	D	E	F	G	H
1	t	y	a1	b1	a2	b2	a3	b3
2	t_1	y_1	=B2*COS(w*A2)	=B2*SIN(w*A2)	=B2*COS(2*w*A2)	=B2*SIN(2*w*A2)	=B2*COS(3*w*A2)	=B2*SIN(3*w*A2)
3	t_2	y_2						
4	t_3	y_3						
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
n	.	.	.	.	.	.	.	.
n+1	t_n	y_n						
n+2		=SUM(B2:Bn+1)	=SUM(C2:Cn+1)					
n+3		$=\frac{B\,n+2}{n}$	$=\frac{2^*C\,n+2}{n}$					
		a_0	a_1	b_1	a_2	b_2	a_3	b_3

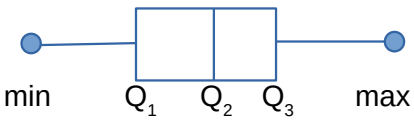
19 Statistics

[K Singh (1) 726-796 9 (2) 806-887]

19.1.1 Notation for Statistics

n	sample size
x x_i	a **sample** statistic (a data value) OR the variate
X	a **population** statistic
$\overline{x}$	the arithmetic mean point of a **sample** set of data
s	standard deviation of a **sample**
μ	the mean value of a **population**
σ	standard deviation of a **population**
$\sum$	the sum of all the terms immediately following
f	frequency (quantity of each data value)
Q	quartile. (Q_1 lower; Q_2 median; Q_3 upper)
d f	degrees of freedom $(n-1)$ of a sample.
$P=(X-x)$	the probability that the population statistic equals the sample statistic
$x!$	$x\times(x-1)\times(x-2)\times(x-3)\times\ldots\times 1,\ \ x\in\mathbb{N}$

Range — maximum value – minimum value 7.1.2



Quartiles — in a set of ordered data,
Median, Q_2 : the middle value.
Lower, Q_1 : the middle value between minimum and Q_2 .
Upper, Q_3 : the middle value between Q_2 and the maximum.
Percentile: the k th percentile is in position $\frac{k}{100}\times n+\frac{1}{2}$.

Mode — in a set of data the mode is the most frequently occurring value.

19.2 Statistical Formulae

Mean, $\bar{x}=\frac{\sum f x}{\sum f}$ or $\bar{x}=\frac{\sum x_i}{n}$ where x_i is the variate,

f is frequency

n is the sample size

BE - averages

Population Standard Deviation $\sigma=\sqrt{\frac{\sum (x_i-\bar{x})^2}{n}}$

$\sigma=\sqrt{\frac{\sum f d^2}{\sum f}}$ $d=x_i-\bar{x}$

Sample Standard Deviation $s=\sqrt{\frac{\sum (x_i-\bar{x})^2}{n-1}}$

or $s=\sqrt{\frac{\left(\sum x_i^2-\frac{\left(\sum x_i\right)^2}{n}\right)}{n-1}}$

where n is the sample size

19.2.1.1 Table for the calculation of Sample Mean and Standard Deviation

x_i	f	$f x_i$	$\lvert x-\bar{x}\rvert$	$f(x-\bar{x})^2$
.	.	.	.	.
.	.	.	.	.
		$\sum f x_i=$		$\sum f(x-\bar{x})^2=$
		$\bar{x}=\frac{\sum f x_i}{n}=$		$s=\sqrt{\frac{\sum f(x-\bar{x})^2}{n-1}}=$

Coefficient of Variation

of a sample (as a %) $\frac{s}{\bar{x}}\times 100$

Semi-interquartile Range $SIR=\frac{Q_3-Q_1}{2}$

19.2.2 Regression Line

- see 9 and 9.1.1

For the line $y=a+bx$ where b is the gradient and a is the y intercept and n is the number of pairs of values.

$$a=\frac{\sum y-b\sum x}{n} \qquad b=\frac{n\sum xy-\sum x\sum y}{n\sum x^2-\left(\sum x\right)^2}$$

Product moment coefficient of Correlation (r value)

$$r=\frac{\left(n\sum xy-\sum x\sum y\right)}{\sqrt{\left(\left(n\sum x^2-\left(\sum x\right)^2\right)\left(n\sum y^2-\left(\sum y\right)^2\right)\right)}} \qquad -1\le r\le 1$$

19.2.2.1 Regression Calculations using a Calculator

Data

Input **X**	x_1	x_2	x_3	.....	x_n
Output **Y**	y_1	y_2	y_3	.....	y_n

Casio fx-83 & 85 GT					**X**	**Y**	
MODE	1	2*	2	1	$x_1=$	$y_1=$	
	COMP	STAT	A+BX	2	$x_2=$	$y_2=$	
			6	3	$x_3=$	$y_3=$	
			A.B^X	:	:	:	
			7	n	$x_n=$	$y_n=$	AC
			A.X^B		► ▲		

SHIFT	1	5*	1	=	SHIFT	1	5*	2	=	SHIFT	1	5*	3	=	MODE	1
	STAT	REG		**A**		STAT	REG		**B**		STAT	REG		**r**		COMP

Casio fx-991ES Replace 2* with 3 and 5* with 7.

19.2.3 Z Scores

$$Z=\frac{x-\mu}{\sigma}$$

19.2.4 Poisson Distribution

- the probability of the occurrence of a rare event

Geogbra Poisson slider

$$P(X=x)=\frac{e^{-\mu}\mu^{x}}{x!}$$

19.2.5 T Test

1 sample

Standard Error of the Mean $$SE(\bar{x})=\frac{s}{\sqrt{n}}$$

T test (1 sample test) $$t=\frac{x-\mu}{SE(\bar{x})}$$

- -

2 sample for $n>30$ (d f = n_1+n_2-2)

Standard Error of Mean $$SE(\bar{x}_1-\bar{x}_2)=\sqrt{\frac{s_1}{n_1}+\frac{s_2}{n_2}}$$

T test (2 sample test) $$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{SE(\bar{x}_1-\bar{x}_2)}$$

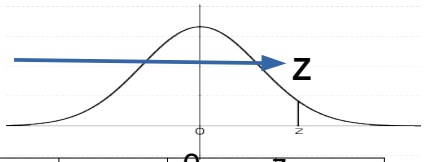
- -

2 sample for $n<30$

Pooled Standard Deviation $$s_p=\sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$$

Standard Error of Mean $$SE(\bar{x}_1-\bar{x}_2)=s_p\sqrt{\frac{1}{n_1}+\frac{2}{n_2}}$$

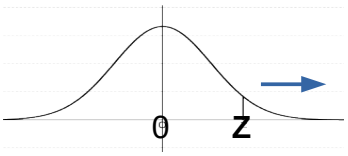
19.2.6 Statistical Tables



19.2.6.1 Normal Distribution

Probability Content from $-\infty$ to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9990	0.9990	1



19.2.6.2 Far Right Tail Probabilities

Z	P{Z to ∞ }	Z	P{Z to ∞ }	Z	P{Z to ∞ }	Z	P{Z to ∞ }
2.0	0.02275	3.0	0.001350	4.0	0.00003167	5.0	2.867E-7
2.1	0.01786	3.1	0.0009676	4.1	0.00002066	5.5	1.899E-8
2.2	0.01390	3.2	0.0006871	4.2	0.00001335	6.0	9.866E-10
2.3	0.01072	3.3	0.0004834	4.3	0.00000854	6.5	4.016E-11
2.4	0.00820	3.4	0.0003369	4.4	0.000005413	7.0	1.280E-12
2.5	0.00621	3.5	0.0002326	4.5	0.000003398	7.5	3.191E-14
2.6	0.004661	3.6	0.0001591	4.6	0.000002112	8.0	6.221E-16
2.7	0.003467	3.7	0.0001078	4.7	0.000001300	8.5	9.480E-18
2.8	0.002555	3.8	0.00007235	4.8	7.933E-7	9.0	1.129E-19
2.9	0.001866	3.9	0.00004810	4.9	4.792E-7	9.5	1.049E-21

These tables are public domain. http://www.math.unb.ca/~knight/utility/NormTble.htm
They are produced by APL programs written by the author, William Knight

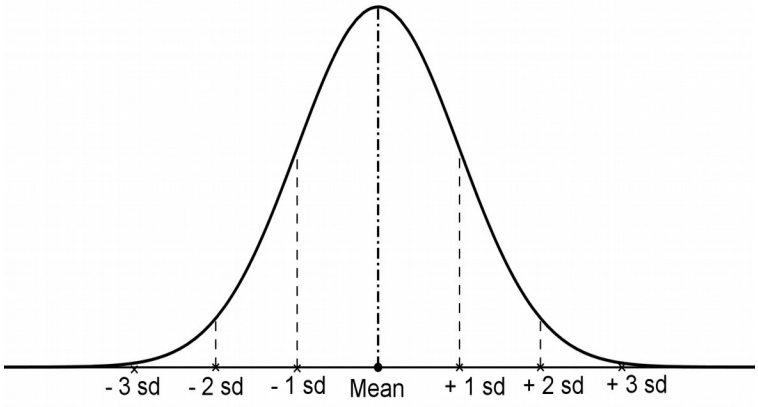
	2-tailed testing				1-tailed testing		
df	**0.1**	**0.05**	**0.01**		**0.1**	**0.05**	**0.01**
5	2.015	2.571	4.032		1.476	2.015	3.365
6	1.943	2.447	3.707		1.440	1.943	3.143
7	1.895	2.365	3.499		1.415	1.895	2.998
8	1.860	2.306	3.355		1.397	1.860	2.896
9	1.833	2.262	3.250		1.383	1.833	2.821
10	1.812	2.228	3.169		1.372	1.812	2.764
11	1.796	2.201	3.106		1.363	1.796	2.718
12	1.782	2.179	3.055		1.356	1.782	2.681
13	1.771	2.160	3.012		1.350	1.771	2.650
14	1.761	2.145	2.977		1.345	1.761	2.624
15	1.753	2.131	2.947		1.341	1.753	2.602
16	1.746	2.120	2.921		1.337	1.746	2.583
17	1.740	2.110	2.898		1.333	1.740	2.567
18	1.734	2.101	2.878		1.330	1.734	2.552
19	1.729	2.093	2.861		1.328	1.729	2.539
20	1.725	2.086	2.845		1.325	1.725	2.528
21	1.721	2.080	2.831		1.323	1.721	2.518
22	1.717	2.074	2.819		1.321	1.717	2.508
23	1.714	2.069	2.807		1.319	1.714	2.500
24	1.711	2.064	2.797		1.318	1.711	2.492
25	1.708	2.060	2.787		1.316	1.708	2.485
26	1.706	2.056	2.779		1.315	1.706	2.479
27	1.703	2.052	2.771		1.314	1.703	2.473
28	1.701	2.048	2.763		1.313	1.701	2.467
29	1.699	2.045	2.756		1.311	1.699	2.462
30	1.697	2.042	2.750		1.310	1.697	2.457
40	1.684	2.021	2.704		1.303	1.684	2.423
50	1.676	2.009	2.678		1.299	1.676	2.403
60	1.671	2.000	2.660		1.296	1.671	2.390
80	1.664	1.990	2.639		1.292	1.664	2.374
100	1.660	1.984	2.626		1.290	1.660	2.364
120	1.658	1.980	2.617		1.289	1.658	2.358
140	1.645	1.960	2.576		1.282	1.645	2.327

2 sample test d f = $(n_1-1)+(n_2-1)=n_1+n_2-2$

19.2.7 Normal Distribution Curve

$$y=\frac{1}{\sigma\sqrt{2\pi}}e^{\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)}$$

$\pm 1\,\text{sd}\approx 68\,\%$
$\pm 2\,\text{sd}\approx 95\,\%$
$\pm 3\,\text{sd}\approx 99.7\,\%$



Geogebra Normal Dist slider
Geogebra Skewed Dist

19.2.8 Binomial Theorem

MC

$$(x+y)^n=\sum_{k=0}^{n}\binom{n}{k}x^{n-k}y^k \qquad \text{where} \quad \binom{n}{k}=\frac{n!}{k!(n-k)!}$$

$$(x+y)^n=x^n+\frac{n!}{1!(n-1)!}x^{n-1}y^1+\frac{n!}{2!(n-2)!}x^{n-2}y^2+\ldots+\frac{n!}{(n-1)!1!}x^1y^{n-1}+y^n$$

19.2.9 Permutations and Combinations

The number of ways of selecting r objects from a total of n

BE - permutations and combinations

Permutations

Repetition allowed	${}^nP_r=n^r$	order does matter
No repetition	${}^nP_r=\frac{n!}{(n-r)!}$	order does matter

Combinations

No repetition	${}^nC_r=\frac{n!}{r!(n-r)!}$	order doesn't matter
Repetition allowed	${}^nC_r=\frac{(n+r-1)!}{r!(r-1)!}$	order doesn't matter

---

Thanks to Gillian Cunningham

20 Financial Mathematics

Notation for Financial Mathematics

i	Interest rate (per time period) expressed as a fraction. (usually written as r)
d	Discount rate (per time period) expressed as a fraction.
n	Number of time periods (sometimes written as i)
P	Principal
A	Accrued amount
a	Amount
S_n	Sum to the n th term (of a geometric progression)
NPV	Net Present Value (of an accrued amount)
irr	Internal Rate of Return (when $NPV = 0$)

Financial Mathematics Formulae

$$r = 1 + i$$

$$A = P(1+i)^n \qquad A = P(1-d)^n$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{a(1 - r^n)}{1 - r}$$

(annuities) $$P = \frac{a(1 - r^{-n})}{r - 1}$$

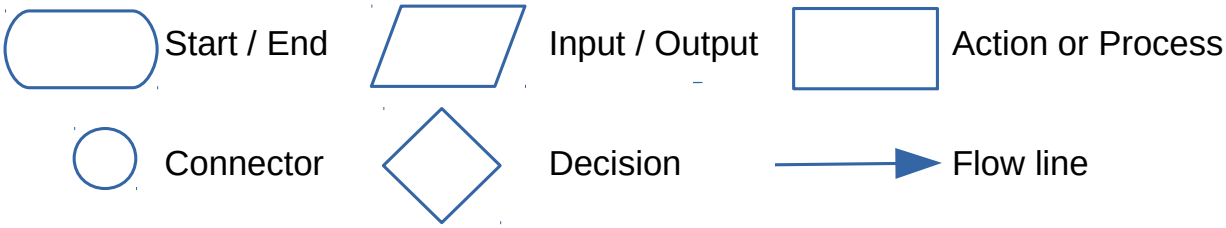
BE - visual guide to interest rates Efunda Calculator

21 Maths for Computing

a_n	a to the base n				
a_{10}	decimal; denary	(a d)	a_2	binary	(a b)
a_{16}	hexadecimal	(a h)	a_8	octal	(a o)

10^3	(1000)	kilo		2^{10}	(1024)	kilobyte	see 29
10^6		Mega		2^{20}	(1024^2)	megabyte	
10^9		Giga	but	2^{30}	(1024^3)	gigabyte	
10^{12}		Tera		2^{40}	(1024^4)	terabyte	
10^{15}		Peta		2^{50}	(1024^5)	petabyte	

21.1.1 Basic Flowchart Shapes and Symbols



22 Mechanical Engineering

[K Singh(1) 2–98 especially 32 – 40 and 69 - 73 (2) 2-99]

22.1.1 Dynamics: Terms and Equations

Linear			Angular		
$s=$	displacement	(m)	θ =	angular displacement	(rad)
$u=$	initial velocity	(m/s)	$\omega_1=$	initial velocity	(rad/s)
$v=$	final velocity	(m/s)	$\omega_2=$	final velocity	(rad/s)
$a=$	acceleration	(m/s^2)	α =	acceleration	(rad/s^2)
$t=$	time	(s)	t =	time	(s)

- -

22.1.2 Conversions

Displacement $s=r\theta$

Velocity $v=r\omega$ $\quad v=\frac{s}{t}$ $\quad \omega=\frac{\theta}{t}$

Acceleration $a=r\alpha$

2π radians = 1 revolution = 360^o, i.e. $1\,rad=\left(\frac{360}{2\pi}\right)^o\approx 57.3^o$ see 14.4.1

If N = rotational speed in revolutions per minute (rpm), then $\omega=\frac{2\pi N}{60}$ rad/s

- -

22.2 Equations of Motion

Linear	Angular
$v=u+at$	$\omega_2=\omega_1+\alpha t$
$s=\frac{1}{2}(u+v)t$	$\theta=\frac{1}{2}(\omega_1+\omega_2)t$
$s=ut+\frac{1}{2}at^2$	$\theta=\omega_1 t+\frac{1}{2}\alpha t^2$
$v^2=u^2+2as$	$\omega_2^2=\omega_1^2+2\alpha\theta$
$a=\frac{v-u}{t}$	$\alpha=\frac{(\omega_2-\omega_1)}{t}$

MC

$\theta=\frac{s}{r}$

$\omega=\frac{v}{r}$

$\alpha=\frac{a}{r}$

- -

Newton's Laws of motion (from Wikipedia - edited)

First law: An object either remains at rest or continues to move at a constant velocity, unless acted upon by a force.

Second law: The vector sum of the forces **F** on an object is equal to the mass *m* of that object multiplied by the acceleration **a** of the object: **F** = *m***a**.

Third law: When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.

$v=\int a\,dt=at+c$ at $t=0, v=u$

$s=\int v\,dt=\int at+u\,dt=\frac{at^2}{2}+ut+c$ at $t=0, s=0$ usually.

22.3 Newton's Second Law

Linear	Angular
$\sum F = ma$	$\sum T = I\alpha$
	where $T = Fr$, $I = mk^2$ and k = radius of gyration

22.3.1 Centrifugal and Centripetal Forces

Definitions from Latin
fugere – to flee
petere – to seek

$$CF = \frac{mv^2}{r}$$

$$CF = m\omega^2 r$$

22.4 Work done and Power

	Linear	Angular
Work Done	$WD = Fs$	$WD = T\theta$
Power	$P = \frac{\text{Work done}}{\text{Time taken}} = \frac{Fs}{t} = Fv$	$P = T\omega$

22.5 Energy

	Linear	Angular
Kinetic Energy	$KE = \frac{1}{2}mv^2$	$KE = \frac{1}{2}I\omega^2$ $KE = \frac{1}{2}mk^2\omega^2$
Potential Energy	$PE = mgh$	

KE of a rolling wheel = KE (linear) + KE (angular)

22.6 Momentum / Angular Impulse

Impulse = Change in momentum MC

Linear	Angular
$Ft = m_2v - m_1u$	$Tt = I_2\omega_2 - I_1\omega_1$

If the mass does not change: $Ft = mv - mu$

Conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

22.7 Specific force / torque values

Force to move a load: $F = \mu m g \cos\theta + m g \sin\theta + m a$

force needed *to overcome* *friction* *weight* *to accelerate*

Force to hoist a load vertically $(\theta = 90^o)$ $F = m g + m a = m(g + a)$

Force to move a load along a horizontal surface $(\theta = 0^o)$ $F = \mu m g + m a$

Winch drum torque $T_{app} = T_F + F_r + I\alpha$

22.8 Stress and Strain

Stress $\sigma = \frac{\text{Load}}{\text{Area}}$ $\sigma = \frac{F}{A}$

Strain $\varepsilon = \frac{\text{change in length}}{\text{original length}}$ $\varepsilon = \frac{\Delta l}{l}$ or $\varepsilon = \frac{x}{l}$

Modulus of Elasticity $E = \frac{\text{Stress}}{\text{Strain}}$ $E = \frac{\sigma}{\varepsilon}$ (Young's Modulus)

Bending of Beams $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

2nd Moment of Area (rectangle) $I = \frac{b d^3}{12}$ *(Area moment of inertia)*

Including the Parallel axis Theorem $I = \frac{b d^3}{12} + A h^2$

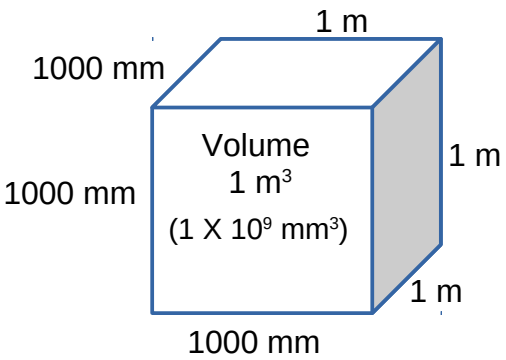
2nd Moment of Area (cylinder) $I = \frac{\pi D^4}{64} - \frac{\pi d^4}{64}$

Torsion Equation $\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$

Thermal Expansion $\Delta L = \alpha L (T_2 - T_1)$

Thanks to Frank McClean, Scott Smith and William Livie

to 5.1.2



All edges length 1 m, (1 X 10^3 mm)

All sides area 1 m^2, (1 X 10^6 mm^2)

$1\frac{MN}{m^2} = 1\frac{N}{mm^2}$ or $1\,MN\,m^{-2} = 1\,N\,mm^{-2}$

22.9 Thermofluids

$$gz_1+\frac{1}{2}C_1^2+h_1+q=gz_2+\frac{1}{2}C_2^2+h_2+w$$

$$z_1+\frac{C_1^2}{2g}+\frac{p_1}{\rho g}=z_2+\frac{C_2^2}{2g}+\frac{p_2}{\rho g}$$

$$\dot{m}=\rho A C$$

$$\dot{V}=\frac{\dot{m}}{\rho}$$

$$p=\rho g z$$

$$v_x=x\,v_g$$

$$u_x=u_f+x\left(u_g-u_f\right)$$

$$h_x=h_f+xh_{fg}$$

$$a^2=b^2+c^2-2bc\cos(A)$$

Thermodynamic non-flow processes summary for m kg of an ideal gas:
For **all** ideal gas, non flow processes:

$$Q=\Delta U+W$$

$$pV=mRT$$

$$\gamma=\frac{c_p}{c_v}$$

$$R=c_p-c_v$$

Process	p, V, T *relationship*	$\Delta U, (U_2-U_1)$ *change in internal entropy*	W *work done*	Q *heat transfer*
Constant volume	$\frac{p_1}{T_1}=\frac{p_2}{T_2}$	$mc_v(T_2-T_1)$	0	$mc_v(T_2-T_1)$
Constant pressure	$\frac{V_1}{T_1}=\frac{V_2}{T_2}$	$mc_v(T_2-T_1)$	$p(V_2-V_1)$ or $mR(T_2-T_1)$	$mc_p(T_2-T_1)$
Constant temperature	$p_1V_1=p_2V_2$	0	$p_1V_1\ln\left(\frac{V_2}{V_1}\right)$ or $mRT_1\ln\left(\frac{V_2}{V_1}\right)$	$p_1V_1\ln\left(\frac{V_2}{V_1}\right)$ or $mRT_1\ln\left(\frac{V_2}{V_1}\right)$
Polytropic	$\frac{p_1}{p_2}=\left(\frac{V_2}{V_1}\right)^n=\left(\frac{T_1}{T_2}\right)^{\frac{n}{n-1}}$	$mc_v(T_2-T_1)$	$\frac{p_1V_1-p_2V_2}{n-1}$ or $\frac{mR(T_1-T_2)}{n-1}$	$W\left(\frac{\gamma-n}{\gamma-1}\right)$
Adiabatic	$\frac{p_1}{p_2}=\left(\frac{V_2}{V_1}\right)^\gamma=\left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$	$mc_v(T_2-T_1)$	$\frac{p_1V_1-p_2V_2}{\gamma-1}$ or $\frac{mR(T_1-T_2)}{\gamma-1}$ or $mc_v(T_1-T_2)$	0

Volumetric flow rate $\dot{V}=A\,v$

Actual flow for a venturi-meter and orifice plate $\dot{V}_{actual}=C_dA_1\left(\frac{2gh\left(\frac{\rho_m}{\rho}-1\right)}{\left(\frac{A_1}{A_2}\right)^2-1}\right)^{\frac{1}{2}}$

22.10 Fluid Mechanics

Volumetric flow rate $Q = A\,v$

Actual flow for a venturi-meter and orifice plate $Q_{actual} = C_d\,A_1\left(\frac{2\,g\,h\left(\frac{\rho_m}{\rho}-1\right)}{\left(\frac{A_1}{A_2}\right)^2-1}\right)^{\frac{1}{2}}$ **Efunda Calculator**

Reynold's Number video

Reynold's number $\mathrm{Re} = \frac{\rho\,v\,d}{\mu}$ $\mathrm{Re} = \frac{v\,d}{\gamma}$ **Efunda calculator**

Darcy formula for head loss $h = \frac{4\,f\,l\,v^2}{2\,g\,d}$

22.11 Heat Transfer

$$\dot{q} = \frac{\Delta T}{R_T}$$

Composites $R_T = \frac{1}{\alpha_A} + \sum \frac{x_n}{\lambda_n} + \frac{1}{\alpha_B}$

Pipe $R_T = \frac{1}{2\pi\,r_{inside}\,\alpha_A} + \sum \frac{\ln\left(\frac{r_n}{r_{n-1}}\right)}{2\pi\,\lambda_n} + \frac{1}{2\pi\,r_{outside}\,\alpha_B}$

Heat Exchanger $\Delta T = \frac{\Delta t_1 - \Delta t_2}{\ln\left(\frac{\Delta t_1}{\Delta t_2}\right)}$

Thanks to Amanda McCartney

22.12 Mechanical Reference

22.12.1 Properties of Materials

Material	Melting point °C	Density ρ kg/m³	Notes
Water (Fresh) (H_2O)	0	1000	at Standard Temperature and Pressure
Plastics		850 - 1400	for PETE/ PVC, polypropylene
Aluminium (Al)	660	2700	at STP
Steel (Fe+C)	1525-1540	7850	at STP
Copper (Cu)	1085	8960	at room temp
Zinc (Zn)	420	7140	at room temp
White Tin (Sn)	232	7365	at room temp
Silver (Ag)	962	10500	at STP
Lead (Pb)	327	11340	at room temp
Mercury (Hg)	-39	13546	at STP
Tungsten (W)	3422	19300	at STP
Gold (Au)	1064	19320	at STP

http://en.wikipedia.org/wiki/Density http://en.wikipedia.org/wiki/Melting_points_of_the_elements_(data_page)

22.12.2 Young's Modulus- approximate.

Material	Young's Modulus GN/m²
Rubber	0.01 – 0.1
HDPE	0.8
Nylon	2 - 4
MDF	4
High strength concrete under compression	30
Aluminium	69
Glass	50 - 90
Brass / Bronze	100 - 125
Titanium alloy	105 - 120
Carbon Fibre reinforced plastic (70/30)	181
Steel	200
Silicon Carbide	450
Tungsten Carbide	450 - 650
Single-walled carbon nanotube	1000+
Diamond	1220

http://en.wikipedia.org/wiki/Young%27s_modulus

22.12.3 Approximate coefficients of friction

Materials		Static friction, μ_s	
		Dry and clean	Lubricated
Aluminium	Steel	0.61	
Copper	Steel	0.53	
Brass	Steel	0.51	
Cast iron	Copper	1.05	
Cast iron	Zinc	0.85	
Concrete	Rubber	1.0	0.30 (wet)
Concrete	Wood	0.62	
Copper	Glass	0.68	
Glass	Glass	0.94	
Metal	Wood	0.2–0.6	0.2 (wet)
Polythene	Steel	0.2	0.2
Steel	Steel	0.80	0.16
Steel	PTFE (Teflon)	0.04	0.04
PTFE (Teflon)	PTFE (Teflon)	0.04	0.04
Wood	Wood	0.25–0.5	0.2 (wet)

http://en.wikipedia.org/wiki/Friction

23 Combinational Logic

$A+0=A$ $\quad$ $A\cdot 0=0$

$A+1=1$ $\quad$ $A\cdot 1=A$

$A\cdot A=A$ $\quad$ $A+A=A$

$A\,\overline{A}=0$ $\quad$ $A+\overline{A}=1$

$\overline{\overline{A}}=A$

$A\cdot B=B\cdot A$ $\quad$ $A+B=B+A$

$A\cdot(B+C)=(A\cdot B)+(A\cdot C)$

$A+(B\cdot C)=(A+B)\cdot(A+C)$

$A\cdot(B\cdot C)=C\cdot(A\cdot B)$ $\quad$ $A+(B+C)=C+(A+B)$

$A\cdot(A+B)=A$ $\quad$ $A+(A\cdot B)=A$

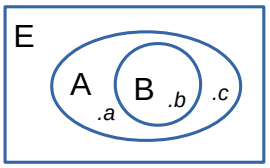
De Morgan's Laws

$\overline{A\cdot B\cdot C\cdot\ldots}=\overline{A}+\overline{B}+\overline{C}+\ldots$ $\quad$ $\overline{A+B+C+\ldots}=\overline{A}\cdot\overline{B}\cdot\overline{C}\cdot\ldots$

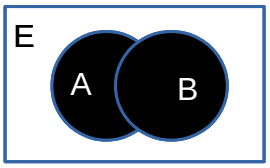
23.1.1 Notation for Set Theory and Boolean Laws

[J Bird pp 377 - 396]

E	universal set
$A=\{a,b,c\ldots\}$	a set A with elements a,b,c etc
$a\in A$	a is a member of A
$\{\ \}$	the empty set ($\varnothing$ is also used)
$B\subset A$	B is a subset of A

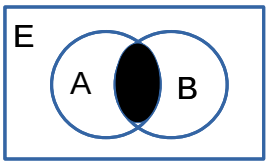


$B\subset A$

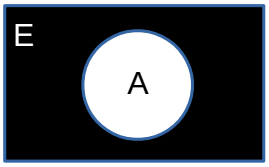


$A\cup B$ $\quad A+B$

Set theory			Boolean	
$\cup$	union	$\vee$	$+$	**OR**
$\cap$	intersection	$\wedge$	$\cdot$	**AND**
A'	complement of A		$\overline{A}$	**NOT**



$A\cap B$ $\quad A\cdot B$



A' $\quad \overline{A}$

24 Electrical Formulae and Constants

Circuit Construction Kit

24.1 Basic

		Unit symbol
Series Resistors	$R_T = R_1 + R_2 + R_3 \ldots.$	Ω
Parallel Resistors	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots.$	Ω
Resistance of Conductor	$R = \frac{\rho \ell}{A}$	Ω
Potential Difference	$V = I R$	V
Power	$P = I V$ or $P = I^2 R$ or $P = \frac{V^2}{R}$	W
Energy (work done)	$W = P t$	J or kWh
Frequency	$f = \frac{1}{T}$	Hz

6

24.2 Electrostatics

Series Capacitors	$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots.$	F
Parallel Capacitors	$C_T = C_1 + C_2 + C_3 \ldots.$	F
Charge	$Q = I t$ or $Q = C V$	C
Capacitance	$C = \frac{A \varepsilon}{d} = \frac{A \varepsilon_0 \varepsilon_r}{d}$	F
Absolute Permittivity	$\varepsilon_0 \approx 8.854 \times 10^{-12}$	F/m

24.3 Electromagnetism

Magnetic Flux Density	$B = \frac{\Phi}{A}$	T
Magnetomotive Force	$F = I N$ or $F = \Phi S$	At
Magnetisation	$H = \frac{I N}{\ell}$	At/m
Reluctance	$S = \frac{l}{\mu A} = \frac{l}{\mu_o \mu_r A}$	At/Wb
Absolute Permeability	$\mu_0 = 4 \pi \times 10^{-7}$ input as $4 \times 10^{(-7)} \times \pi$	H/m

24.4 AC Circuits

			Unit Symbol
Force on a conductor	$F = B I \ell$		N
Electromotive Force	$E = B \ell v$		V
Instantaneous emf	$e = E \sin(\theta)$	$e = E \sin(\omega t)$	V
Induced emf	$e = N \frac{d\varphi}{dt}$	$e = L \frac{di}{dt}$	V
RMS Voltage	$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$	$V_{rms} \approx 0.707\, V_{peak}$	V
Average Voltage	$V_{AV} = \left(\frac{2}{\pi}\right) \times V_{peak}$	$V_{AV} \approx 0.637\, V_{peak}$	V
Angular Velocity	$\omega = 2\pi f$		rad/s 14.4.2
Transformation Ratios	$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$		
Potential Difference	$V = I Z$		V
Power Factor	$\text{pf} = \cos(\varphi)$		
Capacitive Reactance	$X_C = \frac{1}{2\pi f C}$		Ω
Inductive Reactance	$X_L = 2\pi f L$		Ω
Admittance	$Y = \frac{1}{Z}$		S
True Power	$P = V I \cos(\varphi)$		W
Reactive Power	$Q = V I \sin(\varphi)$		VAr
Apparent Power	$S = V I^*$ or $S = P + jQ$		VA

Note: I^* is the complex conjugate of the phasor current. See 13

Thanks to Iain Smith and Jon Procee

24.5 Electrical Reference

24.6 Electrical Tables

Table of Resistivities

Material	Resistivity (ρ) $(\Omega\, m)$ at $20^o\, C$
Silver (Ag)	15.9×10^{-9}
Copper (Cu)	17.2×10^{-9}
Gold (Au)	24.4×10^{-9}
Tungsten (W)	56.0×10^{-9}
Nickel (Ni)	69.9×10^{-9}
Iron (Fe)	100×10^{-9}
Lead (Pb)	220×10^{-9}
Carbon (C)	35000×10^{-9}

Relative Static Permittivity

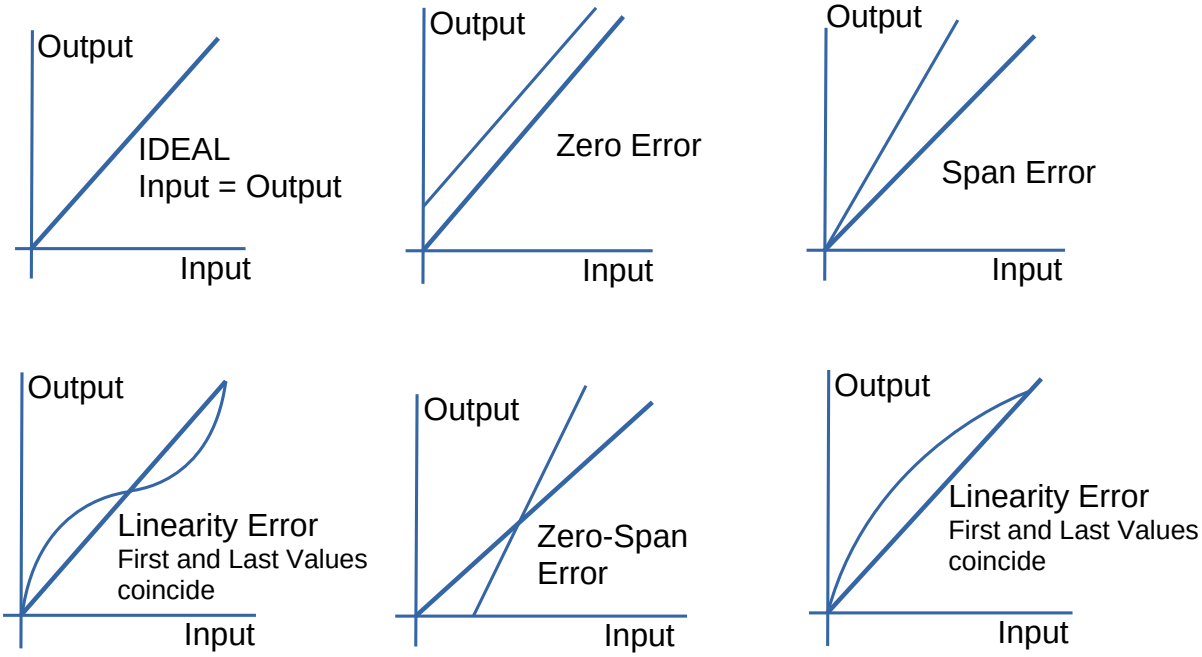
Material	Dielectric Constant ε_r
Vacuum	1
Air	1.00054
Diamond (C)	5.5 - 10
Salt (NaCl)	3 - 15
Graphite (C)	10 - 15
Silicon (Si)	11.68

Permeability Values for some Common Materials

Material	Permeability (μ) (H/m)
Electrical Steel	5000×10^{-6}
Ferrite (Nickel Zinc) (Ni Zn)	$20-800\times10^{-6}$
Ferrite (Manganese Zinc) (Mn Zn)	$>800\times10^{-6}$
Steel	875×10^{-6}
Nickel (Ni)	125×10^{-6}
Aluminium (Al)	1.26×10^{-6}

Thanks to the late Satej Shirodkar

25 Calibration Error



also see 5.1.1

Thanks to Olaniyi Olaosebikan

PID

Proportional/Integral/Differential Control: See https://en.wikipedia.org/wiki/PID_controller
There is a simulator here: https://sites.google.com/site/fpgaandco/pid

Measurement and Uncertainty

National Physical Laboratory. See: http://www.npl.co.uk/publications/uncertainty-guide/uncertainty-of-measurement-guides
UKAS M3003 http://www.ukas.com/library/Technical-Information/Pubs-Technical-Articles/Pubs-List/M3003.pdf

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26 Trig Tables

Sines

Deg	0'	10'	20'	30'	40'	50'	60'	
0	0.0000	0.0029	0.0058	0.0087	0.0116	0.0145	0.0175	89
1	0.0175	0.0204	0.0233	0.0262	0.0291	0.0320	0.0349	88
2	0.0349	0.0378	0.0407	0.0436	0.0465	0.0494	0.0523	87
3	0.0523	0.0552	0.0581	0.0610	0.0640	0.0669	0.0698	86
4	0.0698	0.0727	0.0756	0.0785	0.0814	0.0843	0.0872	85
5	0.0872	0.0901	0.0929	0.0958	0.0987	0.1016	0.1045	84
6	0.1045	0.1074	0.1103	0.1132	0.1161	0.1190	0.1219	83
7	0.1219	0.1248	0.1276	0.1305	0.1334	0.1363	0.1392	82
8	0.1392	0.1421	0.1449	0.1478	0.1507	0.1536	0.1564	81
9	0.1564	0.1593	0.1622	0.1650	0.1679	0.1708	0.1736	80
10	0.1736	0.1765	0.1794	0.1822	0.1851	0.1880	0.1908	79
11	0.1908	0.1937	0.1965	0.1994	0.2022	0.2051	0.2079	78
12	0.2079	0.2108	0.2136	0.2164	0.2193	0.2221	0.2250	77
13	0.2250	0.2278	0.2306	0.2334	0.2363	0.2391	0.2419	76
14	0.2419	0.2447	0.2476	0.2504	0.2532	0.2560	0.2588	75
15	0.2588	0.2616	0.2644	0.2672	0.2700	0.2728	0.2756	74
16	0.2756	0.2784	0.2812	0.2840	0.2868	0.2896	0.2924	73
17	0.2924	0.2952	0.2979	0.3007	0.3035	0.3062	0.3090	72
18	0.3090	0.3118	0.3145	0.3173	0.3201	0.3228	0.3256	71
19	0.3256	0.3283	0.3311	0.3338	0.3365	0.3393	0.3420	70
20	0.3420	0.3448	0.3475	0.3502	0.3529	0.3557	0.3584	69
21	0.3584	0.3611	0.3638	0.3665	0.3692	0.3719	0.3746	68
22	0.3746	0.3773	0.3800	0.3827	0.3854	0.3881	0.3907	67
23	0.3907	0.3934	0.3961	0.3987	0.4014	0.4041	0.4067	66
24	0.4067	0.4094	0.4120	0.4147	0.4173	0.4200	0.4226	65
25	0.4226	0.4253	0.4279	0.4305	0.4331	0.4358	0.4384	64
26	0.4384	0.4410	0.4436	0.4462	0.4488	0.4514	0.4540	63
27	0.4540	0.4566	0.4592	0.4617	0.4643	0.4669	0.4695	62
28	0.4695	0.4720	0.4746	0.4772	0.4797	0.4823	0.4848	61
29	0.4848	0.4874	0.4899	0.4924	0.4950	0.4975	0.5000	60
30	0.5000	0.5025	0.5050	0.5075	0.5100	0.5125	0.5150	59
31	0.5150	0.5175	0.5200	0.5225	0.5250	0.5275	0.5299	58
32	0.5299	0.5324	0.5348	0.5373	0.5398	0.5422	0.5446	57
33	0.5446	0.5471	0.5495	0.5519	0.5544	0.5568	0.5592	56
34	0.5592	0.5616	0.5640	0.5664	0.5688	0.5712	0.5736	55
35	0.5736	0.5760	0.5783	0.5807	0.5831	0.5854	0.5878	54
36	0.5878	0.5901	0.5925	0.5948	0.5972	0.5995	0.6018	53
37	0.6018	0.6041	0.6065	0.6088	0.6111	0.6134	0.6157	52
38	0.6157	0.6180	0.6202	0.6225	0.6248	0.6271	0.6293	51
39	0.6293	0.6316	0.6338	0.6361	0.6383	0.6406	0.6428	50
40	0.6428	0.6450	0.6472	0.6494	0.6517	0.6539	0.6561	49
41	0.6561	0.6583	0.6604	0.6626	0.6648	0.6670	0.6691	48
42	0.6691	0.6713	0.6734	0.6756	0.6777	0.6799	0.6820	47
43	0.6820	0.6841	0.6862	0.6884	0.6905	0.6926	0.6947	46
44	0.6947	0.6967	0.6988	0.7009	0.7030	0.7050	0.7071	45
	60'	50'	40'	30'	20'	10'	0'	Deg

Cosines

Sines

Deg	0'	10'	20'	30'	40'	50'	60'	
45	0.7071	0.7092	0.7112	0.7133	0.7153	0.7173	0.7193	44
46	0.7193	0.7214	0.7234	0.7254	0.7274	0.7294	0.7314	43
47	0.7314	0.7333	0.7353	0.7373	0.7392	0.7412	0.7431	42
48	0.7431	0.7451	0.7470	0.7490	0.7509	0.7528	0.7547	41
49	0.7547	0.7566	0.7585	0.7604	0.7623	0.7642	0.7660	40
50	0.7660	0.7679	0.7698	0.7716	0.7735	0.7753	0.7771	39
51	0.7771	0.7790	0.7808	0.7826	0.7844	0.7862	0.7880	38
52	0.7880	0.7898	0.7916	0.7934	0.7951	0.7969	0.7986	37
53	0.7986	0.8004	0.8021	0.8039	0.8056	0.8073	0.8090	36
54	0.8090	0.8107	0.8124	0.8141	0.8158	0.8175	0.8192	35
55	0.8192	0.8208	0.8225	0.8241	0.8258	0.8274	0.8290	34
56	0.8290	0.8307	0.8323	0.8339	0.8355	0.8371	0.8387	33
57	0.8387	0.8403	0.8418	0.8434	0.8450	0.8465	0.8480	32
58	0.8480	0.8496	0.8511	0.8526	0.8542	0.8557	0.8572	31
59	0.8572	0.8587	0.8601	0.8616	0.8631	0.8646	0.8660	30
60	0.8660	0.8675	0.8689	0.8704	0.8718	0.8732	0.8746	29
61	0.8746	0.8760	0.8774	0.8788	0.8802	0.8816	0.8829	28
62	0.8829	0.8843	0.8857	0.8870	0.8884	0.8897	0.8910	27
63	0.8910	0.8923	0.8936	0.8949	0.8962	0.8975	0.8988	26
64	0.8988	0.9001	0.9013	0.9026	0.9038	0.9051	0.9063	25
65	0.9063	0.9075	0.9088	0.9100	0.9112	0.9124	0.9135	24
66	0.9135	0.9147	0.9159	0.9171	0.9182	0.9194	0.9205	23
67	0.9205	0.9216	0.9228	0.9239	0.9250	0.9261	0.9272	22
68	0.9272	0.9283	0.9293	0.9304	0.9315	0.9325	0.9336	21
69	0.9336	0.9346	0.9356	0.9367	0.9377	0.9387	0.9397	20
70	0.9397	0.9407	0.9417	0.9426	0.9436	0.9446	0.9455	19
71	0.9455	0.9465	0.9474	0.9483	0.9492	0.9502	0.9511	18
72	0.9511	0.9520	0.9528	0.9537	0.9546	0.9555	0.9563	17
73	0.9563	0.9572	0.9580	0.9588	0.9596	0.9605	0.9613	16
74	0.9613	0.9621	0.9628	0.9636	0.9644	0.9652	0.9659	15
75	0.9659	0.9667	0.9674	0.9681	0.9689	0.9696	0.9703	14
76	0.9703	0.9710	0.9717	0.9724	0.9730	0.9737	0.9744	13
77	0.9744	0.9750	0.9757	0.9763	0.9769	0.9775	0.9781	12
78	0.9781	0.9787	0.9793	0.9799	0.9805	0.9811	0.9816	11
79	0.9816	0.9822	0.9827	0.9833	0.9838	0.9843	0.9848	10
80	0.9848	0.9853	0.9858	0.9863	0.9868	0.9872	0.9877	9
81	0.9877	0.9881	0.9886	0.9890	0.9894	0.9899	0.9903	8
82	0.9903	0.9907	0.9911	0.9914	0.9918	0.9922	0.9925	7
83	0.9925	0.9929	0.9932	0.9936	0.9939	0.9942	0.9945	6
84	0.9945	0.9948	0.9951	0.9954	0.9957	0.9959	0.9962	5
85	0.9962	0.9964	0.9967	0.9969	0.9971	0.9974	0.9976	4
86	0.9976	0.9978	0.9980	0.9981	0.9983	0.9985	0.9986	3
87	0.9986	0.9988	0.9989	0.9990	0.9992	0.9993	0.9994	2
88	0.9994	0.9995	0.9996	0.9997	0.9997	0.9998	0.9998	1
89	0.9998	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	0
	60'	50'	40'	30'	20'	10'	0'	Deg

Cosines

Tangents

Deg	0'	10'	20'	30'	40'	50'
0	0.0000	0.0029	0.0058	0.0087	0.0116	0.0145
1	0.0175	0.0204	0.0233	0.0262	0.0291	0.0320
2	0.0349	0.0378	0.0407	0.0437	0.0466	0.0495
3	0.0524	0.0553	0.0582	0.0612	0.0641	0.0670
4	0.0699	0.0729	0.0758	0.0787	0.0816	0.0846
5	0.0875	0.0904	0.0934	0.0963	0.0992	0.1022
6	0.1051	0.1080	0.1110	0.1139	0.1169	0.1198
7	0.1228	0.1257	0.1287	0.1317	0.1346	0.1376
8	0.1405	0.1435	0.1465	0.1495	0.1524	0.1554
9	0.1584	0.1614	0.1644	0.1673	0.1703	0.1733
10	0.1763	0.1793	0.1823	0.1853	0.1883	0.1914
11	0.1944	0.1974	0.2004	0.2035	0.2065	0.2095
12	0.2126	0.2156	0.2186	0.2217	0.2247	0.2278
13	0.2309	0.2339	0.2370	0.2401	0.2432	0.2462
14	0.2493	0.2524	0.2555	0.2586	0.2617	0.2648
15	0.2679	0.2711	0.2742	0.2773	0.2805	0.2836
16	0.2867	0.2899	0.2931	0.2962	0.2994	0.3026
17	0.3057	0.3089	0.3121	0.3153	0.3185	0.3217
18	0.3249	0.3281	0.3314	0.3346	0.3378	0.3411
19	0.3443	0.3476	0.3508	0.3541	0.3574	0.3607
20	0.3640	0.3673	0.3706	0.3739	0.3772	0.3805
21	0.3839	0.3872	0.3906	0.3939	0.3973	0.4006
22	0.4040	0.4074	0.4108	0.4142	0.4176	0.4210
23	0.4245	0.4279	0.4314	0.4348	0.4383	0.4417
24	0.4452	0.4487	0.4522	0.4557	0.4592	0.4628
25	0.4663	0.4699	0.4734	0.4770	0.4806	0.4841
26	0.4877	0.4913	0.4950	0.4986	0.5022	0.5059
27	0.5095	0.5132	0.5169	0.5206	0.5243	0.5280
28	0.5317	0.5354	0.5392	0.5430	0.5467	0.5505
29	0.5543	0.5581	0.5619	0.5658	0.5696	0.5735
30	0.5774	0.5812	0.5851	0.5890	0.5930	0.5969
31	0.6009	0.6048	0.6088	0.6128	0.6168	0.6208
32	0.6249	0.6289	0.6330	0.6371	0.6412	0.6453
33	0.6494	0.6536	0.6577	0.6619	0.6661	0.6703
34	0.6745	0.6787	0.6830	0.6873	0.6916	0.6959
35	0.7002	0.7046	0.7089	0.7133	0.7177	0.7221
36	0.7265	0.7310	0.7355	0.7400	0.7445	0.7490
37	0.7536	0.7581	0.7627	0.7673	0.7720	0.7766
38	0.7813	0.7860	0.7907	0.7954	0.8002	0.8050
39	0.8098	0.8146	0.8195	0.8243	0.8292	0.8342
40	0.8391	0.8441	0.8491	0.8541	0.8591	0.8642
41	0.8693	0.8744	0.8796	0.8847	0.8899	0.8952
42	0.9004	0.9057	0.9110	0.9163	0.9217	0.9271
43	0.9325	0.9380	0.9435	0.9490	0.9545	0.9601
44	0.9657	0.9713	0.9770	0.9827	0.9884	0.9942

Tangents

Deg	0'	10'	20'	30'	40'	50'
45	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295
46	1.0355	1.0416	1.0477	1.0538	1.0599	1.0661
47	1.0724	1.0786	1.0850	1.0913	1.0977	1.1041
48	1.1106	1.1171	1.1237	1.1303	1.1369	1.1436
49	1.1504	1.1571	1.1640	1.1708	1.1778	1.1847
50	1.1918	1.1988	1.2059	1.2131	1.2203	1.2276
51	1.2349	1.2423	1.2497	1.2572	1.2647	1.2723
52	1.2799	1.2876	1.2954	1.3032	1.3111	1.3190
53	1.3270	1.3351	1.3432	1.3514	1.3597	1.3680
54	1.3764	1.3848	1.3934	1.4019	1.4106	1.4193
55	1.4281	1.4370	1.4460	1.4550	1.4641	1.4733
56	1.4826	1.4919	1.5013	1.5108	1.5204	1.5301
57	1.5399	1.5497	1.5597	1.5697	1.5798	1.5900
58	1.6003	1.6107	1.6212	1.6319	1.6426	1.6534
59	1.6643	1.6753	1.6864	1.6977	1.7090	1.7205
60	1.7321	1.7437	1.7556	1.7675	1.7796	1.7917
61	1.8040	1.8165	1.8291	1.8418	1.8546	1.8676
62	1.8807	1.8940	1.9074	1.9210	1.9347	1.9486
63	1.9626	1.9768	1.9912	2.0057	2.0204	2.0353
64	2.0503	2.0655	2.0809	2.0965	2.1123	2.1283
65	2.1445	2.1609	2.1775	2.1943	2.2113	2.2286
66	2.2460	2.2637	2.2817	2.2998	2.3183	2.3369
67	2.3559	2.3750	2.3945	2.4142	2.4342	2.4545
68	2.4751	2.4960	2.5172	2.5386	2.5605	2.5826
69	2.6051	2.6279	2.6511	2.6746	2.6985	2.7228
70	2.7475	2.7725	2.7980	2.8239	2.8502	2.8770
71	2.9042	2.9319	2.9600	2.9887	3.0178	3.0475
72	3.0777	3.1084	3.1397	3.1716	3.2041	3.2371
73	3.2709	3.3052	3.3402	3.3759	3.4124	3.4495
74	3.4874	3.5261	3.5656	3.6059	3.6470	3.6891
75	3.7321	3.7760	3.8208	3.8667	3.9136	3.9617
76	4.0108	4.0611	4.1126	4.1653	4.2193	4.2747
77	4.3315	4.3897	4.4494	4.5107	4.5736	4.6382
78	4.7046	4.7729	4.8430	4.9152	4.9894	5.0658
79	5.1446	5.2257	5.3093	5.3955	5.4845	5.5764
80	5.6713	5.7694	5.8708	5.9758	6.0844	6.1970
81	6.3138	6.4348	6.5606	6.6912	6.8269	6.9682
82	7.1154	7.2687	7.4287	7.5958	7.7704	7.9530
83	8.1443	8.3450	8.5555	8.7769	9.0098	9.2553
84	9.5144	9.7882	10.0780	10.3854	10.7119	11.0594
85	11.4301	11.8262	12.2505	12.7062	13.1969	13.7267
86	14.3007	14.9244	15.6048	16.3499	17.1693	18.0750
87	19.0811	20.2056	21.4704	22.9038	24.5418	26.4316
88	28.6363	31.2416	34.3678	38.1885	42.9641	49.1039
89	57.2900	68.7501	85.9398	114.5887	171.8854	343.7737

27 Periodic Table of The Elements

	1A	2A	3B	4B	5B	6B	7B	8B	8B	8B	1B	2B	3A	4A	5A	6A	7A	8A
1	1 H 1.008 Hydrogen																	2 He 4.003 Helium
2	3 Li 6.941 Lithium	4 Be 9.012 Beryllium											5 B 10.81 Boron	6 C 12.01 Carbon	7 N 14.01 Nitrogen	8 O 16.00 Oxygen	9 F 19.00 Fluorine	10 Ne 20.18 Neon
3	11 Na 22.99 Sodium	12 Mg 24.31 Magnesium											13 Al 26.98 Aluminium	14 Si 28.09 Silicon	15 P 30.97 Phosphorus	16 S 32.07 Sulphur	17 Cl 35.45 Chlorine	18 Ar 39.95 Argon
4	19 K 39.10 Potassium	20 Ca 40.08 Calcium	21 Sc 44.96 Scandium	22 Ti 47.88 Titanium	23 V 50.94 Vanadium	24 Cr 52.00 Chromium	25 Mn 54.94 Manganese	26 Fe 55.85 Iron	27 Co 58.93 Cobalt	28 Ni 58.69 Nickel	29 Cu 63.55 Copper	30 Zn 65.39 Zinc	31 Ga 69.72 Gallium	32 Ge 72.61 Germanium	33 As 74.92 Arsenic	34 Se 78.96 Selenium	35 Br 79.90 Bromine	36 Kr 83.80 Krypton
5	37 Rb 85.47 Rubidium	38 Sr 87.62 Strontium	39 Y 88.91 Yttrium	40 Zr 91.22 Zirconium	41 Nb 92.91 Niobium	42 Mo 95.94 Molybdenum	43 Tc (98) Technetium	44 Ru 101.1 Ruthenium	45 Rh 102.9 Rhodium	46 Pd 106.4 Palladium	47 Ag 107.9 Silver	48 Cd 112.4 Cadmium	49 In 114.8 Indium	50 Sn 118.7 Tin	51 Sb 121.8 Antimony	52 Te 127.6 Tellurium	53 I 126.9 Iodine	54 Xe 131.3 Xenon
6	55 Cs 132.9 Caesium	56 Ba 137.3 Barium	*57 La 138.9 Lanthanum	72 Hf 178.5 Hafnium	73 Ta 180.9 Tantalum	74 W 183.9 Tungsten	75 Re 186.2 Rhenium	76 Os 190.2 Osmium	77 Ir 192.2 Iridium	78 Pt 195.1 Platinum	79 Au 197.0 Gold	80 Hg 200.6 Mercury	81 Tl 204.4 Thallium	82 Pb 207.2 Lead	83 Bi 209.0 Bismuth	84 Po (209) Polonium	85 At (210) Astatine	86 Rn (222) Radon
7	87 Fr (223) Francium	88 Ra 226.0 Radium	**89 Ac 227.0 Actinium	104 Rf (261) Ruther-fordium	105 Db (262) Dubnium	106 Sg (263) Seaborgium	107 Bh (262) Bohrium	108 Hs (265) Hassium	109 Mt (268) Meitner-ium	110 Ds (269) Darm-stadtium	111 Rg (272) Roentgen-ium	112 Cn (277) Copernic-ium	113 Uut (284) Ununtrium	114 Fl (283) Flerovium	115 Uup (288) Ununpent-ium	116 Lv (289) Livermor-ium	117 Uus (293) Ununsept-ium	118 Uuo (294) Ununoct-ium

* Lanthanide Series	58 Ce 140.1 Cerium	59 Pr 140.9 Praseo-dymium	60 Nd 144.2 Neodymium	61 Pm (145) Promethium	62 Sm 150.4 Samarium	63 Eu 152.0 Europium	64 Gd 157.3 Gadolinium	65 Tb 158.9 Terbium	66 Dy 162.5 Dysprosium	67 Ho 164.9 Holmium	68 Er 167.3 Erbium	69 Tm 168.9 Thulium	70 Yb 173.0 Ytterbium	71 Lu 175.0 Lutetium
** Actinide Series	90 Th 232.0 Thorium	91 Pa 231.0 Protactinium	92 U 238.0 Uranium	93 Np 237.0 Neptunium	94 Pu (244) Plutonium	95 Am (243) Americium	96 Cm (247) Curium	97 Bk (247) Berkelium	98 Cf (251) Californium	99 Es (252) Einsteinium	100 Fm (257) Fermium	101 Md (258) Mendelevium	102 No (259) Nobelium	103 Lr (260) Lawrencium

See the Periodic Table of Videos at http://www.periodicvideos.com/

KEY

Atomic number		1
Symbol		H
Atomic mass	***	1.008
Name of the element		Hydrogen

Based on: Cracolice & Peters, Introductory Chemistry, 2/e, 2004.
Thanks to a student for starting this, Fall 1997.
*** or mass number of most common, most stable, or first-discovered isotope.
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28 THE GREEK ALPHABET

UPPER CASE	lower case	Pronunciation	Examples of use
Α	α	Alpha	angles, angular acceleration
Β	β	Beta	angles
Γ	γ	Gamma	shear strain, heat capacity, kinematic viscosity
Δ	δ	Delta	DIFFERENTIAL, the change in... (Calculus)
Ε	ε	Epsilon	linear strain, permittivity
Ζ	ζ	Zeta	IMPEDANCE, damping ratio
Η	η	Eta	efficiency, viscosity
Θ	θ	Theta	angles, temperature, volume strain
Ι	ι	Iota	inertia
Κ	κ	Kappa	compressibility
Λ	λ	Lambda	wavelength, thermal conductivity, eigenvalues
Μ	μ	Mu	micro (10^{-6}), coefficient of friction, permeability
Ν	ν	Nu	velocity
Ξ	ξ	Xi	damping coefficient
Ο	ο	Omicron	
Π	π	Pi	PRODUCT, 3.141592654...., $C=\pi d$
Ρ	ρ	Rho	density, resistivity
Σ	σ	Sigma	SUM; standard deviation, normal stress
Τ	τ	Tau	shear stress, torque, time constant
Υ	υ	Upsilon	admittance
Φ	φ	Phi	phase angle, flux, potential energy, golden ratio
Χ	χ	Chi	Pearson's χ^2 test , angles
Ψ	ψ	Psi	helix angle (gears)
Ω	ω	Omega	RESISTANCE; angular velocity

29 SI Units - Commonly used prefixes

meaning	multiple	prefix	symbol
$\times 1000000000000000$	$\times 10^{15}$	Peta	P
$\times 1000000000000$	$\times 10^{12}$	Tera	T
$\times 1000000000$	$\times 10^{9}$	Giga	G
$\times 1000000$	$\times 10^{6}$	Mega	M
$\times 1000$	$\times 10^{3}$	kilo	k
$\times 1$	$\times 10^{0}$		
$\div 1000$	$\times 10^{-3}$	milli	m
$\div 1000000$	$\times 10^{-6}$	micro	μ
$\div 1000000000$	$\times 10^{-9}$	nano	n
$\div 1000000000000$	$\times 10^{-12}$	pico	p

see 21